Continuity at a Point

A function f is **continuous at a number a** if $\lim_{x \to a} f(x) = f(a)$

If f is not continuous at a, then a is a point of **discontinuity**.

Implied Conditions for Continuity of f at a:

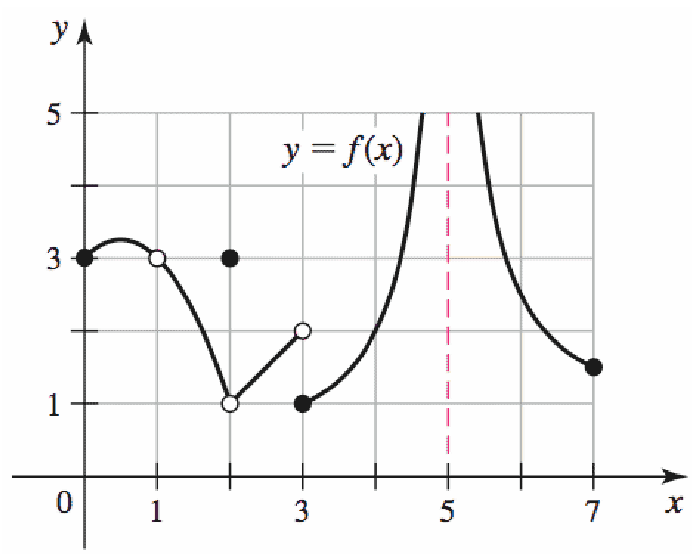
1. f(a) is defined.

2. $\lim_{x \to a} f(x)$ exists

3. $\lim_{x \to a} f(x) = f(a)$

If any item in this list fails to hold, the function fails to be continuous at a.

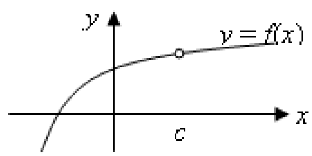
EX: Example 1 Identify values of x on the interval (0, 7) at which f is not continuous.

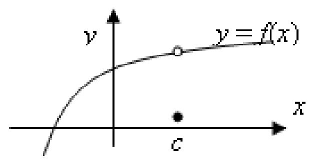


Types of Discontinuity

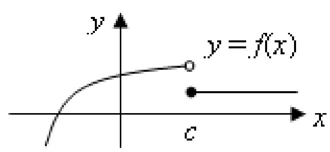
If a function f is not continuous at a number c, we say that f has a discontinuity at c.

1. Removable → We could make the graph continuous (remove the discontinuity) by defining or redefining the function at the input c.



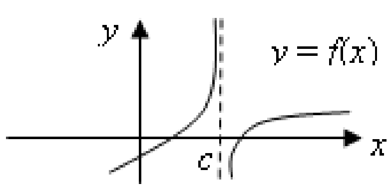


2. Jump → The function jumps from one value to another at c.

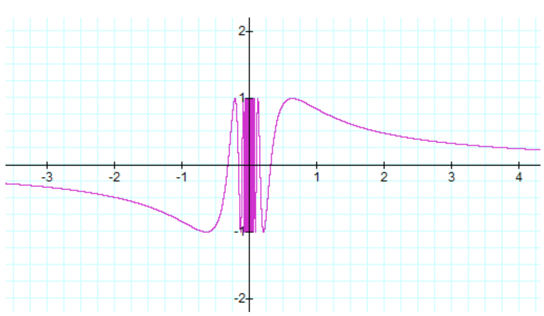


Types of Discontinuity (Continued)

3. Infinite → There is a vertical asymptote at c.



4. Oscillating → The function oscillates too much to have a limit at c.



Continuous Functions

A **continuous function** is continuous at every point of its domain.

NOTE: **Continuity Rules**

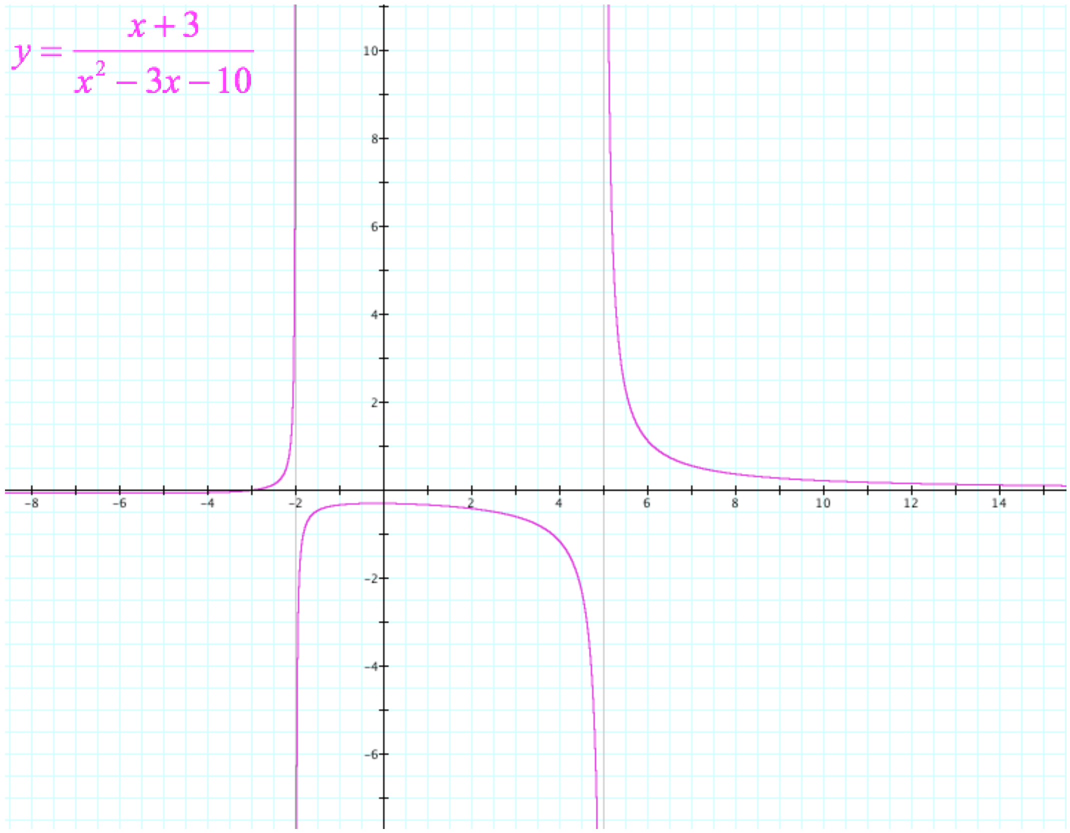
* The inverse of a continuous function is continuous.
* The composition of continuous functions is continuous.

Some Types of Continuous Functions (continuous on their domains)

1. polynomials
2. rational functions
3. root functions
4. trigonometric functions
5. exponential functions
6. logarithmic functions

EX: For what values of x is the function discontinuous?

$$y = \frac{x+3}{x^2 - 3x - 10}$$



<u>Continuity on an Interval</u>

Intuitively, a function is continuous if it is connected; that is, it has no "hole" or break at the point in which we are interested.
(You can draw it without lifting your pencil.)

A function f is continuous from the right at a number a if:

$$\lim_{x \to a^+} f(x) = f(a)$$

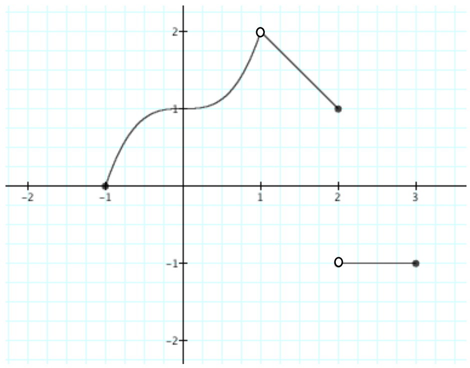
A function f is continuous from the left at a number b if:

$$\lim_{x \to b^-} f(x) = f(b)$$

A function is continuous on an interval if it is continuous at every number in the interval.

(Continuous at an endpoint of an interval is understood to mean continuous from the left or continuous from the right.)

EX: Consider the graph of y = f(x) on its domain [-1, 3].



a. Does f(-1) exist?

b. Does $\lim_{x \to -1^+} f(x)$ exist?

c. Does $\lim_{x \to -1^+} f(x)$ = f(-1)?

d. Is f(x) continuous at x = -1?

e. Where does the function fail to be continuous and why?

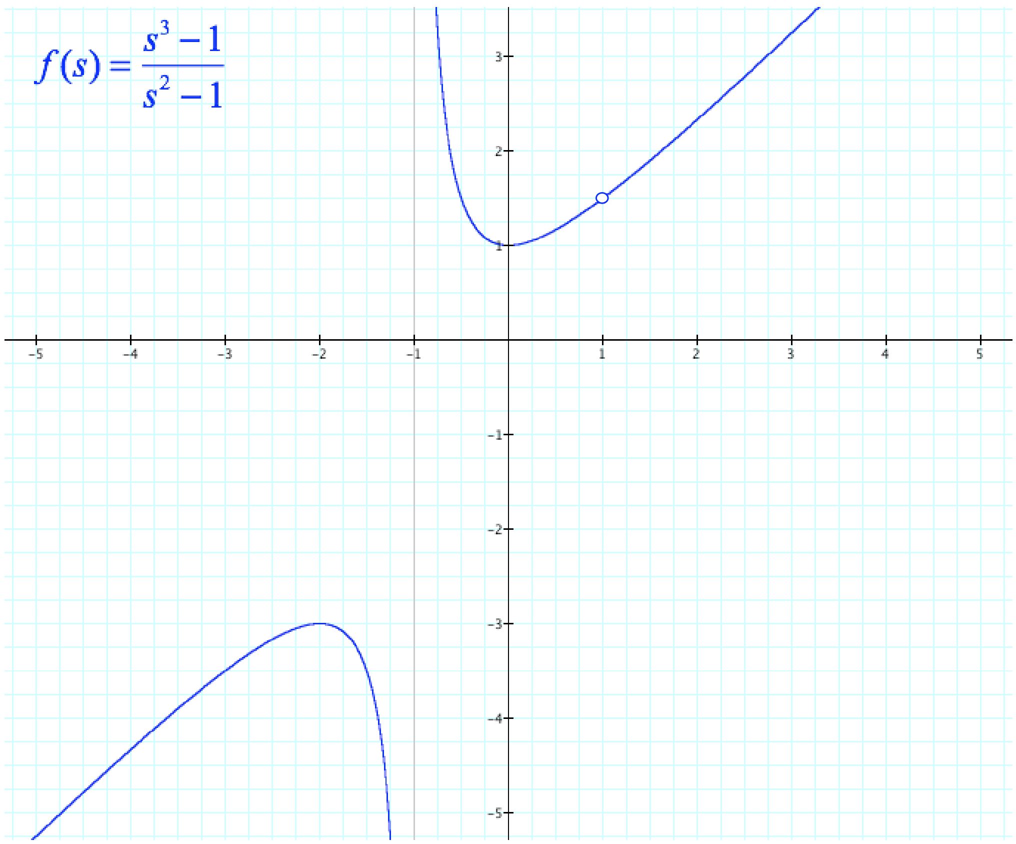
f. On what intervals is f continuous?

g. What value should be assigned to f(1) to make the extended function continuous at x = 1?

EX: Determine the value of the constant a for which the function is continuous at s = 1.

$$f(s) = \begin{cases} \dfrac{s^3 - 1}{s^2 - 1} & \text{if } s \neq 1 \\ a & \text{if } s = 1 \end{cases}$$

$$f(s) = \frac{s^3 - 1}{s^2 - 1}$$

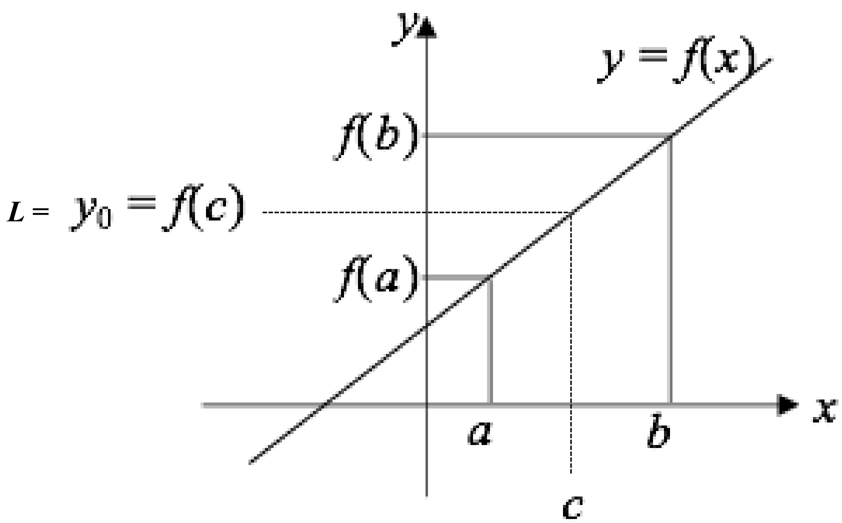


EX: For what value of b is g(x) continuous at every x?

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

The Intermediate Value Theorem

A function f that is continuous on a closed interval [a, b] takes on every value between f(a) and f(b).



EX: Show that the equation, f (x) = $x^4 + x - 3 = 0$, has a solution on the interval [1, 2].

(1) Let's see what the function values are at the endpoints of the interval.

(2) Verify the condition of the IVT is satisfied.

(3) Use the IVT to produce the result.