A Practical Reliability-Based Method for Assessing Soil Liquefaction Potential

Jin-Hung Hwang* and Chin-Wen Yang
Department of Civil Engineering
National Central University
Chung-li, 32054, Taiwan
E-mail: hwangjin@cc.ncu.edu.tw(Hwang)
* Corresponding author

ABSTRACT

The current simplified methods for assessing soil liquefaction potential use a deterministic safety factor to judge whether liquefaction will occur or not. However, these methods are unable to determine the liquefaction probability related to a safety factor. An answer to this problem can be found by reliability analysis. This paper presents a reliability analysis method based on the popular Seed'85 liquefaction analysis method. This method uses the empirical acceleration attenuation law in the Taiwan area to derive the probability density distribution function (PDF) and the statistics for the earthquake-induced cyclic shear stress ratio (CSR). The PDF and the statistics for cyclic resistance ratio (CRR) can be deduced from some probabilistic cyclic resistance curves. These curves are produced by the regression of the liquefaction and non-liquefaction data from the Chi-Chi and other earthquakes around the world, using with minor modifications of the logistic model proposed by Liao *et al*. (1988). The CSR and CRR statistics are used with the first-order and second moment simplified method to calculate the relation of the liquefaction probability with the safety factor and the reliability index. Based on the proposed method, the liquefaction probability related to a safety factor can be easily calculated. The influence of some of soil parameters on the liquefaction probability can be quantitatively evaluated.

Keywords: Soil liquefaction, reliability Analysis, probability density function.

INTRODUCTION

Civil engineers usually use a factor of safety (FS) to evaluate the safeness of a structure. The safety factor is defined as the strength of a member divided by the load applied to it. Most design codes require a member's calculated safety factor should be greater than a specified safety factor, a value larger than one, to ensure the safety of the designed structure. The specified safety factor is largely determined by experience, there has been no rational way to determine such a factor up to now. Because the safety factor-based design method does not consider the variability of the member strength or the applied loading, the probability that the structure will fail can not be known. Engineering design methods based on reliability analysis has been born against this background. The method requires a detailed investigation of the member strength and applied loading data from which statistical indices such as the mean value and variance can be derived. Then, using the first-order and second moment method [1], the relationships between the failure probability, the reliability index and the safety factor can be deduced. As science and technology progress, more data about member strength and loading are collected, making engineering reliability analysis more feasible. These developments have led to the gradual evolution of design codes in various countries [2,3,4], from safety factor-based methods to reliability-based ones.

There has been some research on reliability analysis in liquefaction areas. Haldar and Tang (1979)[5], Fardis and Veneziano(1982)[6], and Chameau and Clough (1983)[7] used the same linear first order and second moment method to assess the variability of the major parameters that influence soil liquefaction and to set up probability models for liquefaction evaluation. However, these models have adopted the early simplified methods for liquefaction evaluation; the soil parameters they used are rarely used now. Moreover, the rationality of the reliability analysis results largely depends on the amount and quality of the collected data used to for deduce the statistics of cyclic earthquake stress and cyclic soil strength. Liao *et al.*(1988)[8] collected data for 289 liquefaction and non-liquefaction cases around the world, then employed the logistic regression model to establish probabilistic cyclic strength curves. Since that effort, this methodology has drawn great attention, and similar probabilistic cyclic resistance curves based on the SPT-N, CPT-q_c, and Vs parameters have been proposed by Youd and Nobel(1997)[9], Toprak *et al.*(1999)[10], and Andrus *et a*l.(2001)[11] respectively. These models only consider the variability of the soil cyclic strength but do not take into consideration the variability of the earthquake-induced cyclic shear stress. Juang et al.(2000a,2000b)[12,13] proposed a limit state curve which separates the states of liquefaction and non-liquefaction by using an artificial neural network, and developed a reliability-based method for assessing the liquefaction potential by introducing the Bayes' mapping theorem. This method has a useful discussion on the relation of the safety factor and the liquefaction probability, which has led to a notable advancement in the state of the art for liquefaction evaluation. Nevertheless, the neural network used with its hidden variables, did not have a clear physical meaning, so that practicing engineers are not very familiar with its use.

In this study, a practical reliability-based method is developed for assessing the soil liquefaction potential. The proposed approach, based on conventional probability theory, enables the earthquake-induced cyclic stress ratio (CSR) and soil cyclic resistance ratio (CRR) statistics to be clearly derived. On the basis of the simplified SPT-N method proposed by Seed *et al.* (1985)[14], the probability density function (PDF) can be deduced for the earthquake-induced cyclic stress ratio, by means of the empirical peak ground acceleration attenuation law and its statistics which are regressed for Taiwan's earthquake data. We used a revised version of the logistic model proposed by Liao *et al*. (1988)[8] to regress the probabilistic cyclic strength curves from cases where liquefaction and non-liquefaction occurred during the

Chi-Chi and other earthquakes around the world. The PDF of the soil cyclic resistance ratio is then derived from these curves. Using the CSR and CRR statistics, it becomes very simple to calculate the relationship of the liquefaction probability and reliability index to a safety factor by way of the first-order and second moment method. A summary of the proposed reliability model, the calculated results, discussion and an application example are given below.

RELIABILITY MODEL FOR SOIL LIQUEFACTION

The first step in engineering reliability analysis is to define the performance function of a structure. If the performance function values of some parts of the whole structure exceed a specified value under a given load, it is thought that the structure will fail satisfy the required function. This specified value (state) is called the limit state of the performance function of the structure. In the simplified liquefaction potential assessment methods, if CSR is denoted as S, and CRR is denoted as R, we can define the performance function for liquefaction as $Z = R - S$. If $Z = R - S < 0$, the performance state is "failure", i.e., liquefaction occurs. If $Z = R - S > 0$, the performance state is "safe", i.e., no liquefaction occurs. If $Z = R - S = 0$, the performance state is on a "limit state", i.e., on the boundary between liquefaction and non-liquefaction states. Since there are some inherent uncertainties in estimating CSR and CRR, we can treat R and S as random variables, hence the liquefaction performance function will also be a random variable. Therefore, the above three performance states can only be assessed to occur with some probability.

The liquefaction probability is the probability that the above inequality will hold. However, an exact calculation of this probability is not easy. In reality, it is difficult to accurately find the $PDFs$ of random variables, R and S. Moreover, the calculation of the probability of the inequality needs multiple integration over the R and S domains, which is a complicated and tedious process.

A simplified calculation method, the first-order and second moment method, has been developed against this background. The method uses statistics for the basic independent random variables, such as R and S, to calculate the approximate statistics of the performance function variable, Z in this case, so as to bypass the complicated integration process. According to the principle of statistics, the performance function $Z = R - S$ is also a normal distribution random variable, if both R and S are independent random variables under normal distribution. If the probability density function (PDF) and cumulative probability function (CPF) of Z are denoted as $f_z(z)$ and $F_z(z)$, respectively. The liquefaction probability P_f then equals the probability of $Z = R - S \leq 0$. Hence

$$P_f = P(Z \leq 0) = \int_{-\infty}^{0} f_z(z)dz = F_z(0) \quad (1)$$

This is shown in Figure 1. If the mean values and standard deviations of R and S are μ_R, μ_S and σ_R, σ_S, according to the first-order and second moment method, the mean value μ_z, the standard deviation σ_z, and the covariance coefficient δ_z of Z can be derived as follows.

$$\mu_Z = \mu_R - \mu_S \quad (2)$$

$$\sigma_z = \sqrt{\sigma_R^2 + \sigma_S^2} \quad (3)$$

$$\delta_Z = \frac{\sigma_Z}{\mu_Z} = \frac{\sqrt{\sigma_R^2 + \sigma_S^2}}{\mu_R - \mu_S} \quad (4)$$

By equations (2), (3) and (4), the statistics for the performance function Z can be simply calculated, using the statistics for the basic variables R and S. This shows the advantage of the first order and second moment method. A reliability index β is defined as the inverse of the covariance coefficient of δ_z, to measure the reliability of the liquefaction evaluation results. β is expressed as

$$\beta = \frac{1}{\delta_z} = \frac{\mu_z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (5)$$

$$\mu_z = \beta\sigma_z \quad (6)$$

In Figure 1 the liquefaction probability is shown by the shaded tail areas of the PDF $f_z(z)$ of the performance function Z. Since $\mu_z = \beta\sigma_z$, the larger the β, the greater the mean value μ_z, and the smaller the shaded area and the liquefaction probability P_f. This means that β has a unique relation with P_f and can be used as an index to measure the reliability of the liquefaction evaluation.

Assuming that R and S are independent variables with a normal distribution, then, $Z = R - S$ is also in a normal distribution of $Z \sim (\mu_z, \sigma_z^2)$. By placing the PDF of Z into Equation (1), we obtain the following liquefaction probability P_f.

$$P_f = \int_{-\infty}^{0} f_z(z)dz = \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{1}{2}(\frac{z-\mu_z}{\sigma_z})^2} dz . \quad (7)$$

The above equation can be rewritten as

$$P_f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{\mu_z}{\sigma_z}} e^{-\frac{t^2}{2}} dz = \Phi(-\frac{\mu_z}{\sigma_z}) \ ; \ t = \frac{z - \mu_z}{\sigma_z}, \quad (8)$$

where Φ is the cumulative probability function for a standard normal distribution. Since $\beta = \mu_z / \sigma_z$, hence

$$P_f = \Phi(-\beta) = 1.0 - \Phi(\beta) . \quad (9)$$

The probability distribution of the basic engineering variables are usually slightly skewed, so they can't be reasonably modeled by a normal distribution function. It has been found that most of the basic variables in engineering areas can be much better described by a log-normal distribution model, such as that proposed by Rosenblueth and Estra (1972)[15]. In this research we also assume that R and S are log-normal distributions. Based on this assumption, the reliability index β and the liquefaction probability P_f can be expressed as

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_{\ln R} - \mu_{\ln S}}{\sqrt{\sigma^2_{\ln R} + \sigma^2_{\ln S}}} = \frac{\ln\left[\frac{\mu_R}{\mu_S}\left(\frac{\delta_S^2+1}{\delta_R^2+1}\right)^{1/2}\right]}{\left[\ln(\delta_R^2+1)(\delta_S^2+1)\right]^{1/2}} \tag{10}$$

$$P_f = \Phi(-\beta) = 1.0 - \Phi(\beta) \tag{11}$$

According to the safety factor-based design method, the safety factor FS for liquefaction is defined as the ratio of the mean values of R and S. Hence

$$FS = \frac{\mu_S}{\mu_R} \tag{12}$$

By putting (12) into (10), we can obtain a one to one relation of reliability index β or liquefaction probability P_f with a safety factor FS for the given coefficients of variance δ_R and δ_S. Thus, the liquefaction probability P_f corresponding to any given safety factor FS can be derived.

PROBABILITY DENSITY FUNCTION OF CYCLIC STRESS RATIO

The most widely used simplified SPT-N method is that proposed by Seed et al. (1985), hereafter, referred to as the Seed'85 method. This method calculates the earthquake-induced cyclic stress ratio in a soil layer via the simplified equation below.

$$CSR = 0.65 \cdot \frac{\sigma'_V}{\sigma_V} \cdot \frac{A_{\max}}{g} \cdot r_d(z) / MSF(M), \tag{13}$$

where σ'_V, σ_V are the effective and total vertical overburden pressures at some specified depth; $A_{\max}$ is the peak horizontal ground acceleration; $r_d(z)$ is the stress reduction factor at depth z, $MSF(M)$ is a magnitude scaling factor that considers the duration effect of different earthquake magnitudes. In equation (13), σ'_V, σ_V are directly computed from boring log and laboratory test data, and can therefore be regarded as deterministic values with no variance; The $r_d(z)$ and $MSF(M)$ vary with the depth z and the earthquake magnitude M. Both these values have significant variability, however, reliable statistics have not yet been obtained. Hence, at present we can only calculate the $r_d(z)$ and $MSF(M)$ factors from the deterministic chart and table suggested by Seed and Idriss(1982)[16]. The greatest uncertainty in the CSR is primarily governed by the uncertainty involved in estimating the peak ground acceleration $A_{\max}$ for a given earthquake event with a magnitude M. Therefore, $A_{\max}$ is the dominant factor producing the CSR variance.

The $A_{\max}$ can usually be estimated by the so called empirical acceleration attenuation formula, which shows the attenuated relation of the peak ground acceleration $A_{\max}$ with the epicentral or hypocenter distance R, for an earthquake with a given magnitude M. These attenuation formulae are usually regressed from the measured seismic data for $A_{\max}, M, R$ in some functional forms. Jean(1996)[17] adopted the attenuation formulae that suggested by Campbell(1981)[18] to regress seismic data collected in Taiwan as follows:

$$A_{\max} = 0.0278\exp(1.999M)[R + 0.1413\exp(0.6918M)]^{-1.7347} \tag{14}$$

$$\sigma_{\ln(A_{\max})} = 0.5389\,, \tag{15}$$

where $A_{\max}$ = horizontal peak ground acceleration (g)

R = hypocentral distance (km)

M= local Richter magnitude.

Equation (13) shows that CSR is a linear function of $A_{\max}$. Therefore, Equation (13) can be simplified as $CSR = a \cdot A_{\max}$. This means that the statistics for CSR can be calculated using the $A_{\max}$ statistics as follows:

$$CSR = 0.65 \cdot \frac{\sigma_v}{\sigma_v^{'}} \frac{A_{\max}}{g} r_d = a \cdot A_{\max} \tag{16a}$$

$$\mu_{CSR} = a \cdot \mu_{A\max} \tag{16b}$$

$$\delta_{CSR} = \delta_{A\max} \tag{16c}$$

$$\delta_{A_{\max}} = \sqrt{\exp(\sigma^2_{\ln(A_{\max})}) - 1} \tag{16d}$$

$$\sigma_{\ln(CSR)} = \sqrt{\ln(\delta^2_{CSR} + 1)} \tag{16e}$$

$$\mu_{\ln(CSR)} = \ln(\mu_{CSR}) - 0.5\sigma^2_{\ln(CSR)}\,, \tag{16f}$$

where μ_X, δ_X and σ_X are the mean value, the coefficient of variance and standard deviation of variable X; $\mu_{\ln(X)}, \delta_{\ln(X)}$ and $\sigma_{\ln(X)}$ are the mean value, the coefficient of covariance and the standard deviation of variable $\ln(X)$. If we assume that CSR is a log-normal probability distribution, its density function can be written as

$$f_{CSR}(CSR)=\frac{1}{\sqrt{2\pi}\sigma_{\ln(CSR)}\cdot CSR}\exp[-\frac{1}{2}(\frac{\ln(CSR)-\mu_{\ln(CSR)}}{\sigma_{\ln(CSR)}})^2]\text{ , } 0<x<\infty \quad (17)$$

Figure 2 shows an example of a probability density function calculated for a soil layer at a depth of 10 meters.

PROBABILITY DENSITY FUNCTION OF CYCLIC RESISTANCE RATIO

In the conventional simplified methods, an empirical cyclic resistance curve has been used with some normalized penetration parameters, such as $(N_1)_{60}$ and $CPT-q_c$, to estimate CRR. It is not known how much variability the estimated CRR will have. Based on the liquefaction and non-liquefaction cases collected for this study, we modify Liao *et al.*'s logistic model (1988) to achieve the regression of the CRR probability density function. The authors collected a total of 699 sets of data which included 397 sets of data reported by Loertscher and Youd (1994)[19] and 302 sets of data published by Hwang and Yang (2001)[20].

The logistic model is an important regression method in common use for binary data, such as for liquefaction or non-liquefaction. It was Liao *et al.* (1988) who pioneered the use of a logistic model for the treatment of liquefaction and non-liquefaction data and established a series of probabilistic cyclic resistance curves.

Such curves are valuable for providing probability information about the estimated CRR, however, they can not reveal more rigorously statistical information, such as the mean value and variance, so that they can not be directly used in a reliability analysis.

According to Hwang and Yang (2001)[20], the empirical relation of CRR and $(N_1)_{60}$, i.e., the empirical cyclic resistance curve, can be better expressed by an exponential function. Thus, in this study the following probabilistic curves are used to regress the collected data:

$$P_L = \frac{1}{1 + \exp\{-[\beta_0 + \beta_1 (N_1)_{60} + \beta_2 (N_1)_{60}^2 + \beta_3 \ln(CSR)]\}} \tag{18}$$

where P_L is the liquefaction probability under a set of $(N_1)_{60}$ and CSR, and $\beta_0, \beta_1, \beta_2, \beta_3$ are the parameters to be regressed. Seed *et al*. (1985) have found that for a given $(N_1)_{60}$, CRR increases as fines content increases. They suggest an empirical curve for correcting the $(N_1)_{60}$ of a silty sand to an equivalent standard penetration resistance $(N_1)_{60CS}$ for clean sand. We use this correction curve to convert $(N_1)_{60}$ to $(N_1)_{60CS}$ for our data. The regression results from equation (18) are shown in Table 1 and Figure 3. We can rewrite equation (18) as

$$CSR = \exp[\frac{-\ln(1/P_L - 1) - \beta_0 - \beta_1 (N_1)_{60CS} - \beta_2 (N_1)_{60CS}^2}{\beta_3}] \tag{19}$$

Equation (19) can be interpreted as for a soil with a given $(N_1)_{60CS}$; P_L is the probability that the CRR will be smaller than the CSR induced by an earthquake.

P_L is the liquefaction probability. Also, P_L can be regarded as the cumulative probability that CRR will be less than a specified CSR. Hence

$$P_L = F(CRR) = F(CRR < CSR)\,, \tag{20}$$

where $F(CRR)$ is the cumulative probability function of CRR. Thus, the derivative of equation (20) gives the probability density function of the CRR for a soil with a given $(N_1)_{60CS}$. The probability density function is

$$f(CRR) = \frac{dF(CRR)}{dCRR} = -\frac{ab(CRR)^{b-1}}{(1+a(CRR)^b)^2} \tag{21a}$$

$$a = \exp[-\beta_0 - \beta_1(N_1)_{60CS} - \beta_2(N_1)^2_{60CS}] \tag{21b}$$

$$b=-\beta_3 \tag{21c}$$

where $f(CRR)$ is the probability density function of CRR, An example of the distribution is shown in Figure 4. It indicates that the distribution is near the log-normal one. The mean value and the deviation of the distribution increase as the $(N_1)_{60}$ value increases. This indicates that the estimated CRR is more uncertain for a soil with a high $(N_1)_{60}$ value.

The mean and median values are the two indices used in statistics to show the centralized trend of the sample space. The following equations (22) and (23) show the CRR mean and medium curves respectively. Equation (23) is derived from equation (19) by putting $P_L = 0.5$. The difference between these two curves is shown in Figure 5 (calculated by the Mathematica software). If the curve of equation (19) is compared to the $P_L = 0.6$ curve, it can be seen that this curve almost coincides with the mean curve of the CRR. This means that the probability distribution function of CRR is skew to the right:

$$\text{mean}: E(CRR) = \int CRR \times f(CRR)dCRR \tag{22}$$

$$\text{medium}: CRR = \exp[-\frac{\beta_0}{\beta_3} - \frac{\beta_1}{\beta_3}(N_1)_{60CS} - \frac{\beta_2}{\beta_3}(N_1)^2_{60CS}] \quad (P_L=0.5) \tag{23}$$

$$P_L=0.6: CRR = \exp[-\frac{\ln(1/0.6-1)}{\beta_3} - \frac{\beta_0}{\beta_3} - \frac{\beta_1}{\beta_3}(N_1)_{60CS} - \frac{\beta_2}{\beta_3}(N_1)^2_{60CS}] \tag{24}$$

The mean value and the variance coefficient are the two statistical parameters needed in the first-order and second moment method. However, the numerical integration necessary to calculate the mean value of the CRR as in equation (22), is rather complicated for practical use. In this study, we can use the simpler equation (24), i.e. the probabilistic cyclic resistance cure of $P_L = 0.6$, to approximate the mean value of the CRR with only negligibe error. Although the variance in the CRR increases with the mean, as shown in Figure 4, the calculated variance coefficient δ_{CRR} is a constant value of 0.604. The 95% confidence interval ranges within 0.206～1.579$\overline{CRR}$. The $\overline{CRR}$ can be calculated with equation (24).

LIQUEFACTION PROBABILITY AND SAFETY FACTOR

The largest advantage of a reliability-based liquefaction evaluation analysis is that the liquefaction probability behind a specified safety factor can be quantitatively evaluated. The relation of the liquefaction probability and the safety factor to liquefaction, can be calculated by the simple equation (25) below. Equation (25) is derived by utilizing the statistics for CRR and CSR (see Table 2) in equations (10),(11) and (12).

$$\beta = \frac{\ln\left[\frac{\mu_R}{\mu_S}\left(\frac{\delta_S^2+1}{\delta_R^2+1}\right)^{1/2}\right]}{\left[\ln(\delta_R^2+1)(\delta_S^2+1)\right]^{1/2}} = -0.013 + \frac{\ln(FS)}{0.7758} \qquad (25a)$$

$$P_f = \Phi(-\beta) = 1.0 - \Phi(\beta) \qquad (25b)$$

The whole procedure is outlined in the flow chart in Figure 6. Figure 7 shows the calculated liquefaction probability related to the safety factor, subject to different variance coefficients. It shows that, for the same safety factor, if $FS<1.0$, the greater the coefficient of variance, the higher the liquefaction probability. However, if $FS>1.0$, the greater the coefficient of variance, the lower the liquefaction probability. Therefore, to assess the potential for liquefaction, the variances of CRR and CSR are the more important factors of influence in probability analysis.

The empirical critical cyclic resistance curve, as suggested by the simplified Seed'85 method, usually has some degree of conservativeness. How much is implied by the empirical curve, can be quantified by the proposed probability model. Figure 8 shows a comparison of the probabilistic cyclic resistance curves of P_L=0.2, 0.5 and 0.6 with the empirical curve of the Seed'85 method. We find that each point on the empirical curve has a different liquefaction probability. For reference and comparison, several points on the empirical curve are marked with their corresponding liquefaction probability (calculated using the proposed model). To get the liquefaction probability of the safety factor calculated with the well known Seed'85 method, we denote the safety factor as FS_{Seed}, and develop the relation between FS_{Seed} and FS, as defined with reliability analysis below:

$$FS = \frac{\mu_R}{\mu_S} = \frac{Cr \cdot \mu_{R,Seed}}{\mu_S} = Cr \frac{\mu_{R,Seed}}{\mu_S} = Cr \cdot FS_{Seed} \tag{26a}$$

$$Cr = \frac{\mu_R}{\mu_{R,Seed}} \quad , \tag{26b}$$

where $\mu_{R,Seed}$ is the CRR computed by the Seed'85 method and μ_R is the mean value of the CRR computed with Equation (24). C_r is the ratio of μ_R to $\mu_{R,Seed}$. When the value of $(N_1)_{60}$ is between 8~30, C_r is within the range of 1.18~1.55, with an average value of 1.3, as is shown in Figure 8. The relation of the liquefaction probability P_L with the FS_{Seed}, based on the proposed model, is compared with that suggested by Juang *et al*. (2002)[21]. See Figure 9. It is found that if FS_{Seed}=1.0, the P_L is not 50%, but has a value of 26%~43%. When compared with Juang's relation, we find that, in this study if FS_{Seed}<1.0, the P_L is higher than Juang's. On the other hand, if FS_{Seed} >1.0, the P_L in this study is lower than Juang's. This difference is due to the different sources for seismic and soil data on which these two methods are based. One notable difference is that the coefficient of variance of the *CSR* calculated by Taiwan's acceleration attenuation formula, is larger than that used by Juang *et al*.(2002).

To reach a preliminary understanding of the influence of some of the important parameters on P_L, the $(N_1)_{60}$, fines content FC(%), and the ground water table (G..W.T.) aree chosen for conducting a sensitivity study. Figure 10 shows P_L variations in a soil layer at a depth of 8m, resulting from variations in the above mentioned parameters. Figure 10(a) indicates that P_L decreases significantly as $(N_1)_{60}$ increases. Figure 10(b) shows that P_L decreases slightly as fines content increases for the range of $(N_1)_{60}$ between 10~30. Figure 10(c) indicates that P_L decreases significantly as the ground water table becomes lower, especially in the range of $(N_1)_{60} < 20$.

AN APPLICATION EXAMPLE

An important construction site is planned in Tainan county, the second largest county in south Taiwan. However, the site is located near an active fault, the Hsinhwa fault, which had 12km of surface rupture during the 1946 Tainan earthquake. The Tainan earthquake had a magnitude of M_L=6.3 and caused extensive liquefaction in area surrounding fault. Therefore, a careful assessment of the liquefaction potential of the site is required.

A simplified geological profile is shown in Figure 11. In the profile, there are only two liquefiable sandy soil layers, located at G.L.-8~-14.5m and G.L.-16.5~-19.5m. The water table is 5.3 m below the ground surface. The design earthquake, assessed by seismologists, should be M_L=6.8, which will create a peak horizontal acceleration of 0.28 g at the site. The soil parameters and the results of the liquefaction analysis are shown in Table 3 and Figure 11. They indicate that FS_{Seed} = 0.8 only for soil at a depth near 15m, the safety factors of the other soil layers are all greater than 1.2. Based on the proposed model, the liquefaction probability P_L is 62% for the soil , where FS_{Seed} = 0.8, and ranges from 6% ~35% for the other soil layers , where FS_{Seed} >1.2.

SUMMARY AND DISCUSSION

This paper presents a practical reliability-based method for liquefaction analysis. The proposed method is simple and clear. On the basis of the popular Seed'85 method, the authors use the empirical acceleration attenuation law to derive the probability density distribution function (PDF) and statistics for the earthquake-induced cyclic shear stress ratio (CSR) in Taiwan area. They also collected liquefaction and non-liquefaction data from Chi-Chi and other earthquakes around the world, then, used the logistic model proposed by Liao *et al*. (1986) to derive the PDF and statistics for cyclic resistance ratio (CRR). With these statistics, the first-order and second moment method can be used to calculate the relation of the liquefaction probability to the safety factors and the reliability index. The whole proposed computation procedure is summarized in a flow chart, to facilitate its use by engineers. Finally, an analysis assessing the liquefaction potential at a real construction site, is presented, to demonstrate its use.

In the application example, it is found that even with a safety factor of 1.2, the soil still has a liquefaction probability of about 35% for the given design earthquake. This probability may be considered a little higher at first glance, however, we must note that the liquefaction probability derived in the study does not consider the probability of the occurrence of the given earthquake. It only gives the liquefaction probability for one soil layer, for the given earthquake event. Therefore, the real liquefaction probability would be the joint probability of liquefaction occurrence during an earthquake and the probability of that an earthquake of such a magnitude will occur. Based on the seismic hazard analysis, the probability that the specified earthquake will occur is 0.002 annually, or , in other words, this size earthquake has a return period of 475 years, in Taiwan. Hence, in the above example the real liquefaction probability is considerably less than 0.002, and this seems to be more reasonable. Thus, a complete probabilistic liquefaction analysis method would consider the uncertainties of the CSR and CRR, as well as the probability that an earthquake will occur. This needs further development.

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REFERENCES

[1] Ang AHS, Tang WH. Probability Concepts in Engineering Planning and Design. John Wiley & Sons, New York, 1975.

[2] National Standard, People's Republic of China. (in Chinese). Building Code of Earthquake Resistant Design-GBJ11-89. Beijing, China: The Chinese Building Publishing House, 1989.

[3] AASHTO. STANDARD Specifications for Highway Bridges. 16th Edition, 1996.

[4] Eurocode 8. ENV 1997-1: Geotechnical Design. European Committee for Standardization (CEN) Brussels,1994,.

[5] Haldar A, Tang WH. Probabilistic evaluation of liquefaction potential. J Geotech Engng 1979; 105(2):145-163.

[6] Fardis MN, Veneziano D. Probabilistic analysis of deposit liquefaction. J Geotech Engng 1982; 108(3): 395-417.

[7] Chameau JL, Clough GW. Probabilistic pore pressure analysis for seismic

loading. J Geotech Engng 1983;109(4): 507-524.
[8] Liao SSC, Veneziano D, Whitman RV. Regression models for evaluating liquefaction probability. J Geotech Engng 1988; 114(4): 389-411.
[9] Youd TL, Noble SK. Liquefaction criteria based on statistical and probabilistic analyses. Proc NCEER Workshop, Technical Report NCEER-97-0022 1997: 201-216.
[10] Toprak S, Holzer TL, Bennett MJ, Tinsley JC. CPT- and SPT-based probabilistic assessment of liquefaction. Proc of 7th US-Japan Workshop on Earthquake Resistant Design of Lifeline Facilities and Counter-measures Against Liquefaction, Seattle, August 1999: 69-86.
[11] Andrus RD, Stokoe KH, Chung RM, Juang CH. Guidelines for Evaluation Liquefaction Resistance Using Shear Wave Velocity Measurements and Simplified Procedures. National Institute of Standards and Technology, Gaithersburg, MD 2001.
[12] Juang CH, Chen CJ, Rosowsky DV, Tang WH. CPT-based liquefaction analysis Part 1: Determination of limit state function. Geotechnique 2000a; 50(5): 583-592.
[13] Juang CH, Chen CJ, Rosowsky DV, Tang WH. CPT-based liquefaction analysis Part 2: Reliability for design. Geotechnique 2000b; 50(5): 593-599.
[14] Seed HB, Tokimatsu K, Harder LF, Chung RM. The Influence of SPT Procedures in Soil Liquefaction Resistance Evaluation. J Geotech Engng 1985; 111(12): 1425-1445.
[15] Rosenblueth E, Estra L. Probabilistic design of reinforced concrete buildings. ACI Special Publication1972; 31: 260p.
[16] Seed HB, Idriss IM. Ground Motions and Soil Liquefaction during earthquakes. EERI Monograph 1982.
[17] Jean WY. A study on reliability analysis for structure and design seismic force. Doctoral Thesis, National Taiwan University, Taipei, Taiwan, 1996
[18] Campbell KW. Near-source attenuation of peak horizontal acceleration. BSSA 1981; 71(6): 2039-2070.
[19] Loertscher TW, Youd TL. Magnitude scaling factors for analysis of liquefaction hazard. unpublished Research Report No. CEG. 94-02, Department of Civil and Environ Engng, Brigham Young University, Provo, Utah, 1994.
[20] Hwang JH, Yang CW. Verification of critical cyclic strength curve by Taiwan

Chi-Chi earthquake data. Soil Dynam Earthquake Engng 2001; 21:237-257.

[21] Juang CH. Jiang T, Andrus RD. Assessing probability-based methods for liquefaction potential evaluation. J Geotech and Geoenviron Engng 2002; 128(7):580-589.

Table 1 Parameters in the logistic model

Parameter	β_0	β_1	β_2	β_3
Regressed result	10.4	-0.2283	-0.001927	3.8

Table 2 Mean values and variance coefficients of CSR and CRR

	Mean value	Variance coefficient
CSR	$0.65 \cdot \frac{\sigma_v}{\sigma_v'} \cdot \frac{A_{\max}}{g} \cdot r_d \cdot MSF(M)$	0.581
CRR	$\exp[-2.63 + 0.06008(N_1)_{60} + 0.000507(N_1)_{60}^2]$	0.604

Table 3 Result of liquefaction analysis for the site near the Hsinhwa fault

depth (m)	Unit weight (t/m^3)	SPT-*N*	*FC* (%)	Soil classification	*F.S.* (Seed)	P_L (%)
1.3	1.97	3	73	CL-ML	-	-
2.8	2.02	6	69	CL-ML	-	-
4.3	2.00	7	75	CL-ML	-	-
5.8	1.89	15	82	ML	-	-
7.3	1.93	6	99	ML	-	-
8.8	2.01	6	91	CL-ML	-	-
10.3	1.98	17	33	SM	1.2	35%
11.8	1.95	23	29	SM	1.4	19%
13.3	1.87	18	33	SM	1.2	35%
14.8	1.96	13	14	SM	0.8	62%
16.3	1.95	9	99	CL	-	-
18.8	2.04	33	25	SM	2.0	6%
19.3	2.19	33	20	SM	1.9	9%

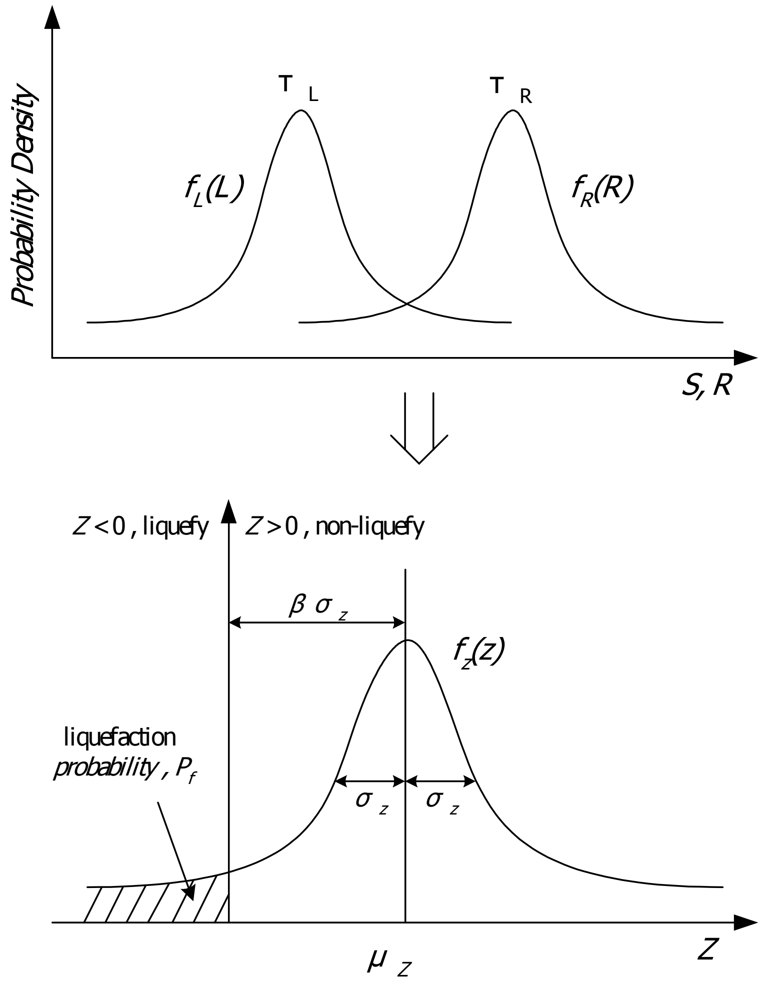

Fig.1 Probability density distribution for the liquefaction performance function.

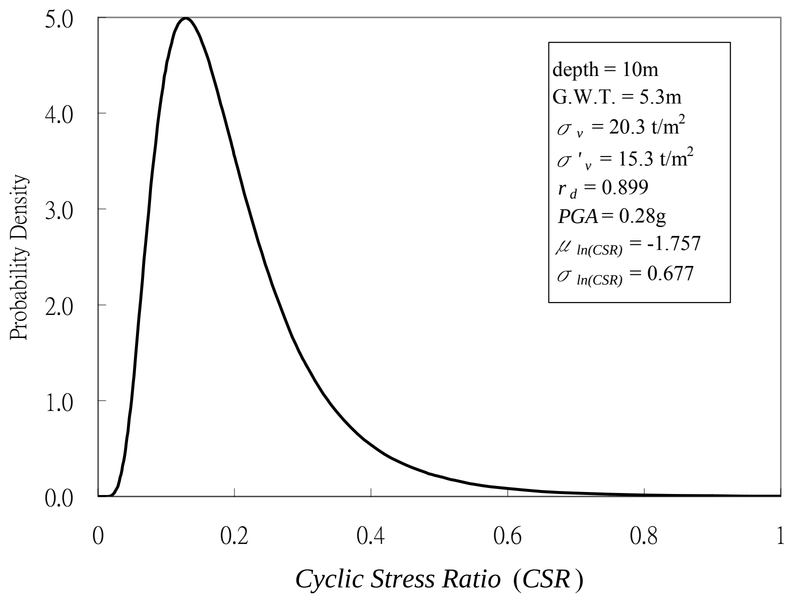

Fig.2 Calculated probability density function of a soil at a depth of 10 m.

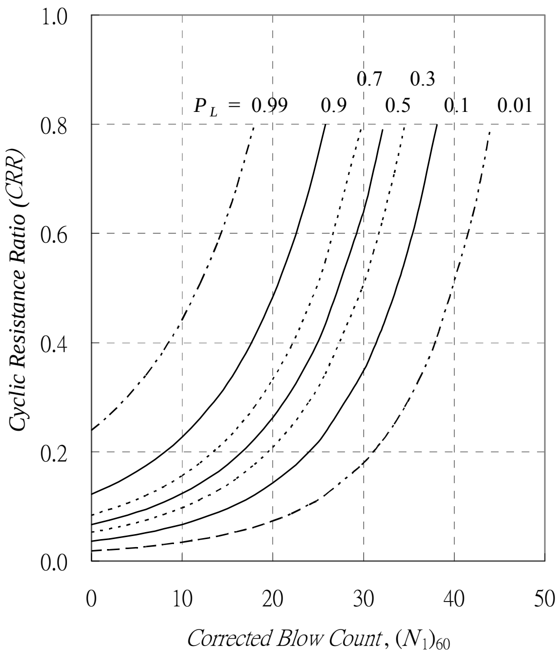

Fig.3 Probabilistic cyclic resistance curves regressed by the logistic model.

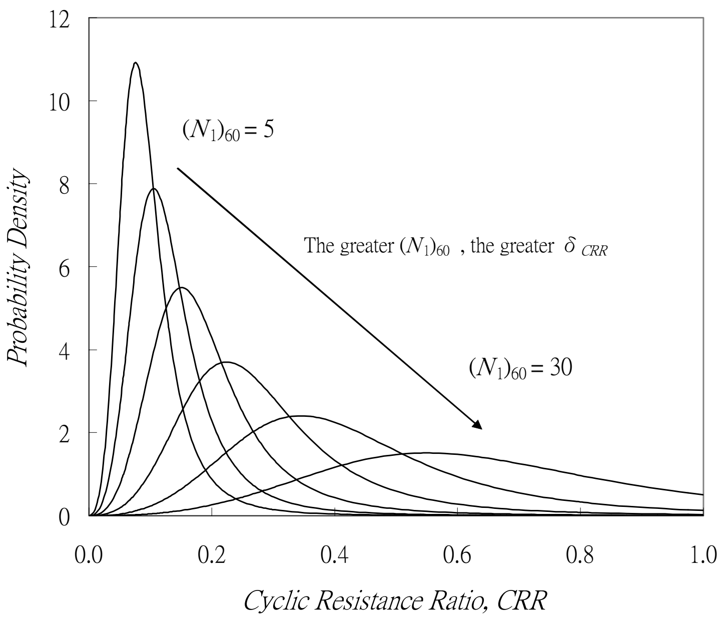

Fig.4 Probability density function of the soil cyclic resistance ratio.

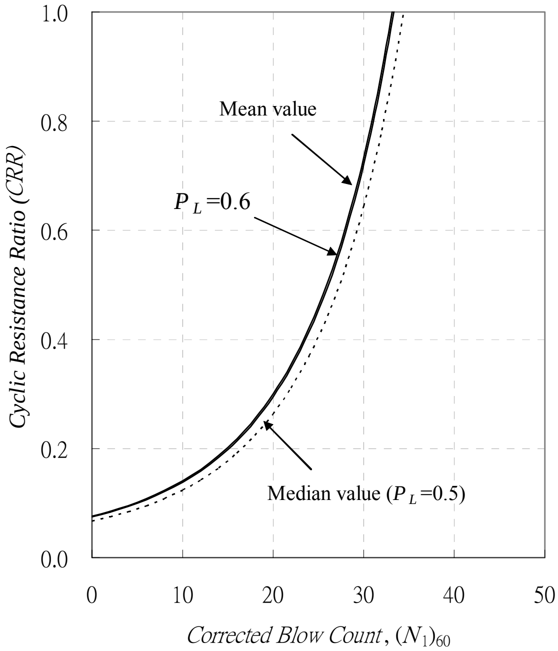

Fig.5 Mean and median curves compared with the probabilistic curve of P_L =0.6.

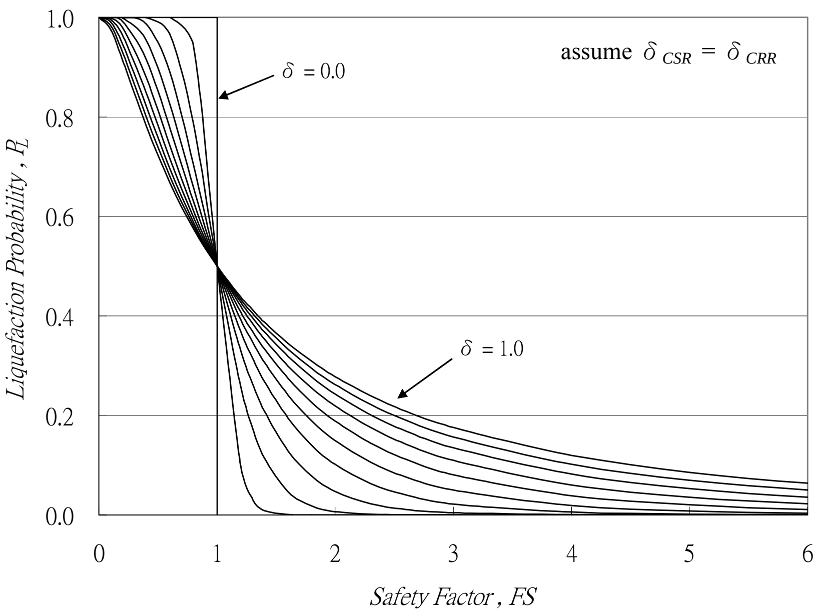

Fig.7 Relations of liquefaction probability with the safety factor for different variance coefficients.

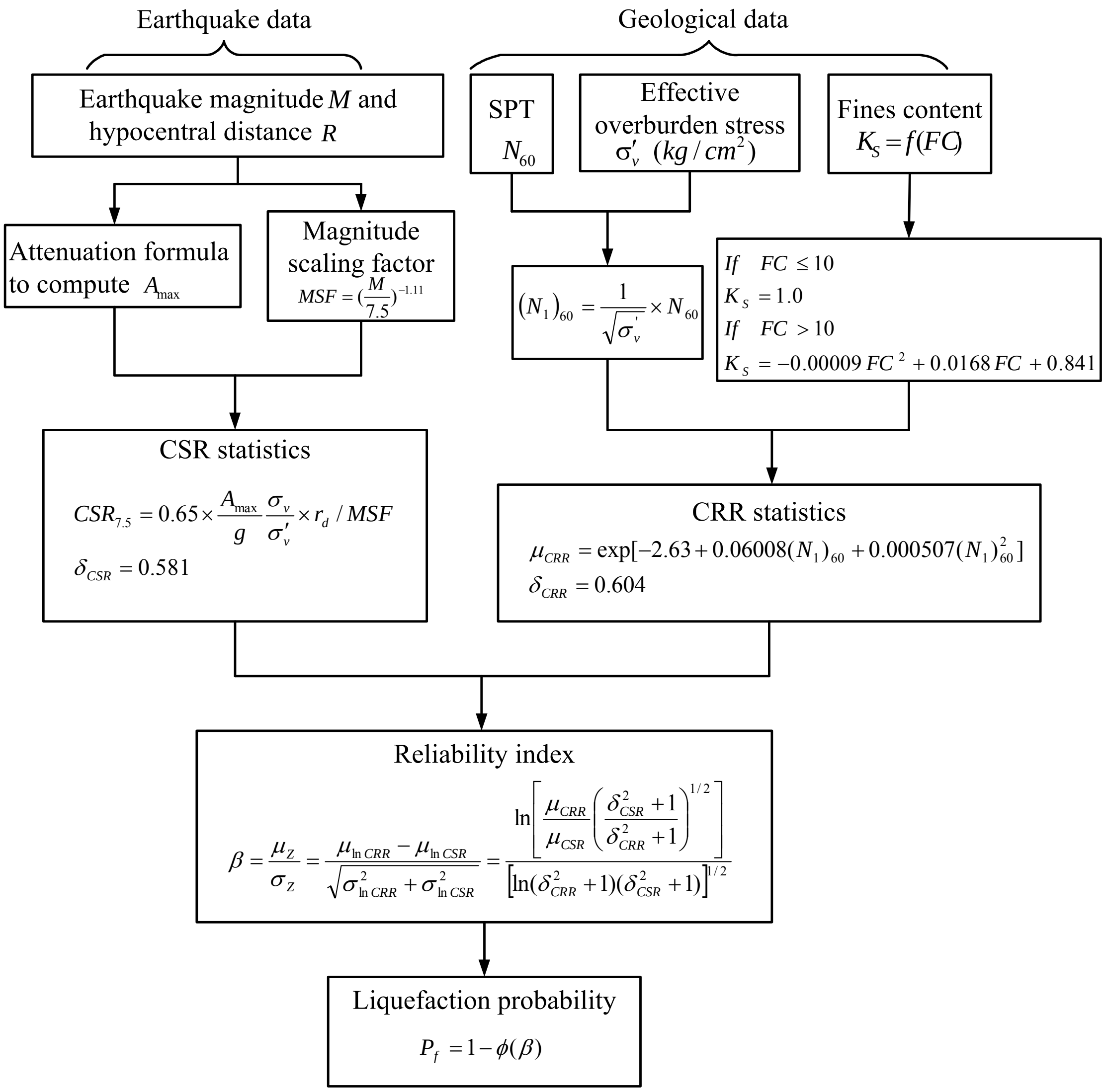

Fig.6 Flow chart of the proposed reliability liquefaction analysis method.

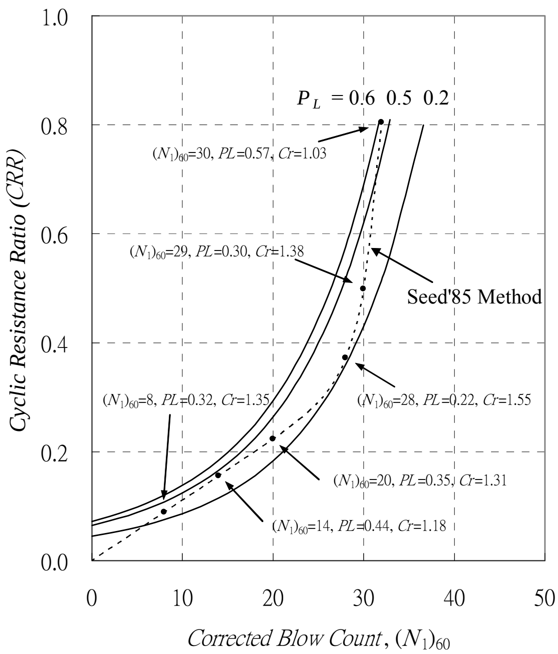

Fig.8 Comparison of the probabilistic CRR curves with the empirical curve proposed by Seed'85 method.

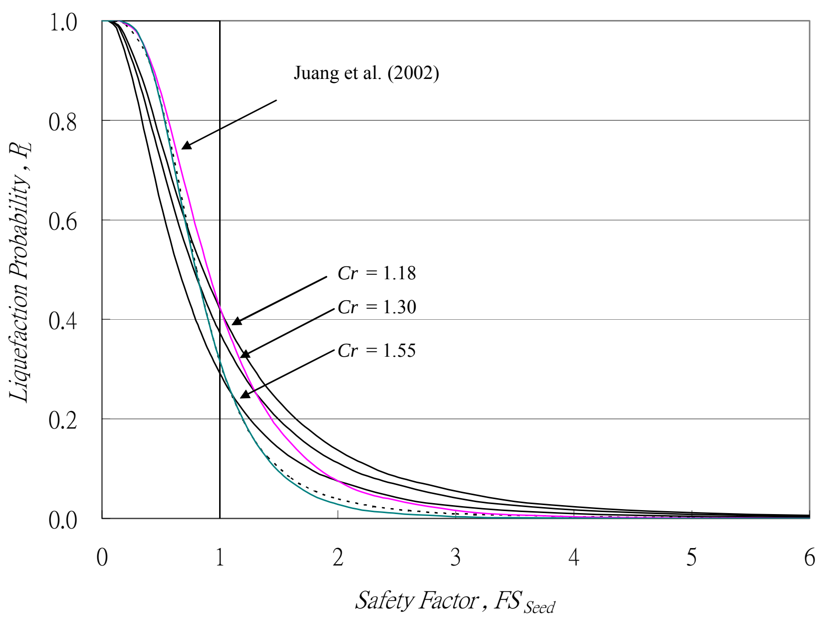

Fig.9 Relation of liquefaction probability with the safety factor calculated by Seed'85 method.

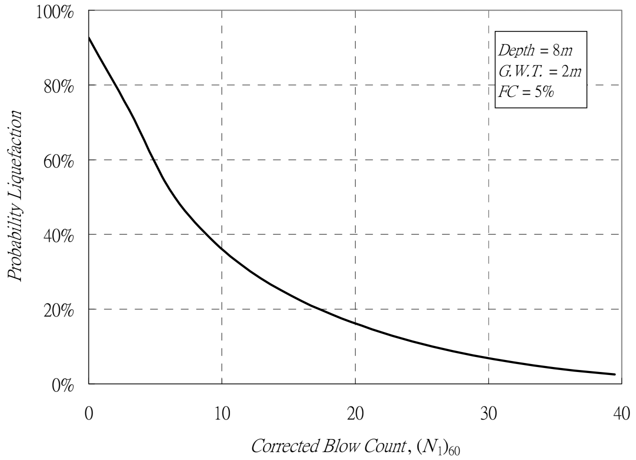

Fig.10(a) Variation of liquefaction probability with $(N_1)_{60}$.

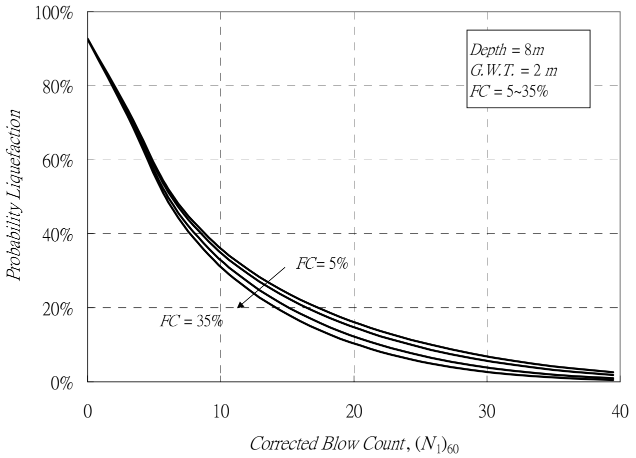

Fig.10(b) Influence of fines content on liquefaction probability.

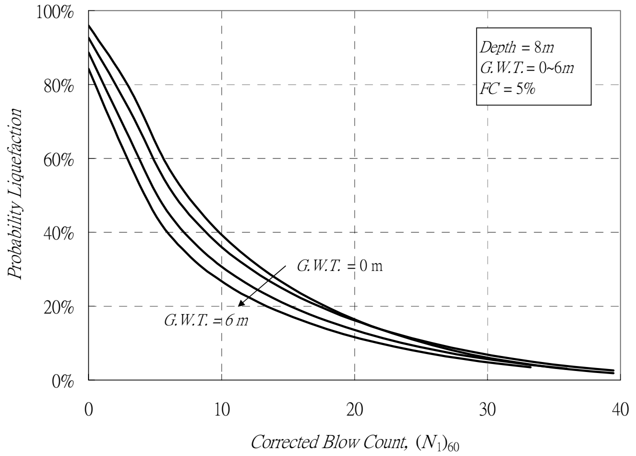

Fig.10(c) Influence of ground water table on liquefaction probability.

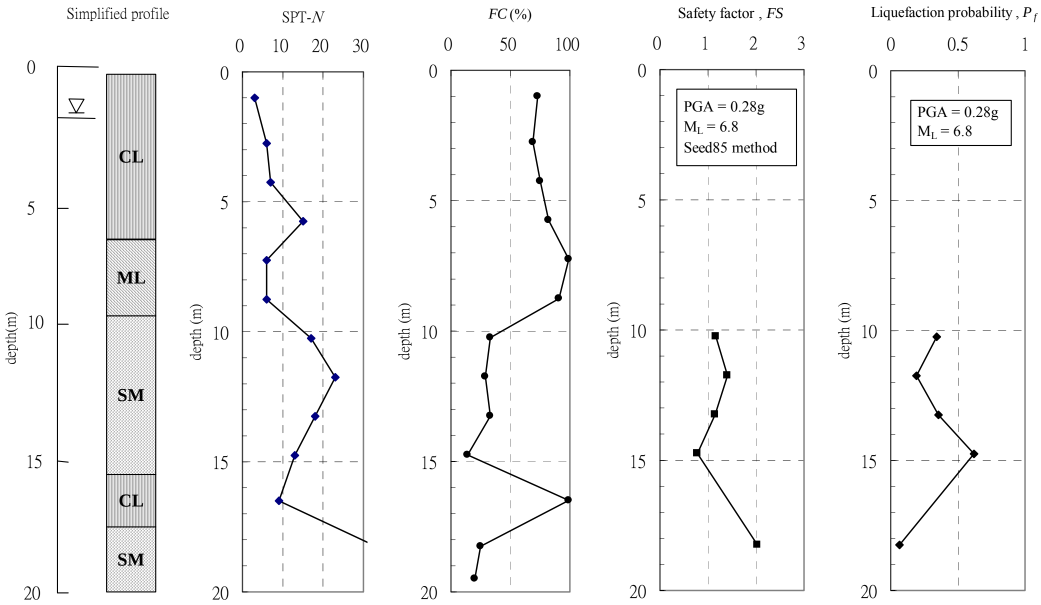

Fig.11 Result of liquefaction analysis for the site near the Hsinhwa fault.