C.2. Nanoscale Positioning and Manipulation through Feedback Tracking Control of Piezoelectrically-driven Nanostages:

In high frequencies, dynamics of the actuator plays an important role in the response of the system. Therefore, feedforward controller does not guarantee the accuracy of the positioning. Here, we introduce a feedback controller utilizing the Sliding Mode Control with Perturbation Estimation (SMCPE) strategy. To design the controller, we modify the equation of motion associate with the actuator to the following equation:

\[ \ddot{x} + 2\xi \omega_n \dot{x} + \omega_n^2 x = H\{v(t)\} + p(t) \tag{6} \]

where \( p(t) \) is the time-varying perturbation function which contains all model perturbations, and is estimated in real-time as

\[ p_{est}(t) = \ddot{x} + 2\xi \omega_n \dot{x} + \omega_n^2 x - H\{v(t - \tau)\} \tag{7} \]

We assume that the actuator displacement, velocity and acceleration are measurable, and input voltage is continuously applied. For the system described by (6), if the control input is given by

\[ v(t) = H^{-1}\{\dot{x}_d + 2\xi \omega_n \dot{x} + \omega_n^2 x + \sigma \dot{e} + \gamma \text{sat}(s) + \lambda s - p_{est}(t)\} \tag{9} \]

where \( \text{sat}(\cdot) \) represents the saturation function, (a continuous approximation of signum function), \( \sigma \), \( \gamma \) and \( \lambda \) are the positive scalars, then asymptotically task-space and subtask tracking of the system is guaranteed in the sense that the signal \( e(t) \) is bounded \( (e(t) \rightarrow 0 \text{ as } t \rightarrow \infty \text{ if } e(t) \text{ is bounded}) \). The detail derivation and the stability proof of the proposed controller are given in the references here. We use the PI direct and inverse hysteresis models that have been previously identified in the feedforward control approach.

The derived controller is implemented on the single Z-axis nanostager for different low- and high-, and multiple-frequency desired trajectories. Tracking control results are shown in Figure 10. The profile of the desired trajectories and the maximum and mean square values of the tracking error are given in Table 1. Tracking results indicates that controller is able to accurately track the desired trajectories in lower frequencies. However, achieving high speed multiple-frequency trajectory control with small tracking error enables fast and accurate positioning, scanning, and manipulating objects through scanning probe microscopes, using the proposed controller.

<table>
<thead>
<tr>
<th>Desired trajectory profile (µm)</th>
<th>Maximum error (%)</th>
<th>Mean square error (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 3 - 3 cos(20πt)</td>
<td>1.24</td>
<td>0.03</td>
</tr>
<tr>
<td>(b) 4 - [cos(2πt) + cos(6πt) + cos(10πt) + cos(20πt)]</td>
<td>0.87</td>
<td>0.03</td>
</tr>
<tr>
<td>(c) 3 - 3 cos(100πt)</td>
<td>2.91</td>
<td>0.11</td>
</tr>
<tr>
<td>(d) 4 - [cos(20πt) + cos(30πt) + cos(80πt) + cos(100πt)]</td>
<td>2.74</td>
<td>0.07</td>
</tr>
<tr>
<td>(e) 3 - 3 cos(200πt)</td>
<td>5.32</td>
<td>0.19</td>
</tr>
<tr>
<td>(f) 4 - [cos(60πt) + cos(100πt) + cos(140πt) + cos(200πt)]</td>
<td>4.23</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Figure 10: Trajectory control results: (a) 10Hz sinusoidal, (b) low-speed multi-frequency, (c) 50Hz sinusoidal, (d) medium-speed multi-frequency, (e) 100Hz sinusoidal, and (f) High-speed multi-frequency trajectories.
This subtask has resulted in the following publications during this report period.

