Use the method of separation of variables to show that the 1-D heat conduction problem: find $u = u\ x,y$ so that

$$\frac{\partial u}{\partial t}(x,t\ - \alpha \frac{\partial^2 u}{\partial x^2}(x,t\ = 0,\ 0 < x < l,\ t > 0$$

$$u(0,t\ = 0,\ t \geq 0$$

$$u(l,t\ = 0,\ t \geq 0$$

$$u(x,0\ = f(x\ ,\ 0 \leq x \leq l$$

has the solution

$$u(x,t\ = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} \exp\left[-\alpha \left(\frac{n\pi}{l} \right)^2 t \right]$$

where

$$c_n = \frac{2}{l} \int_0^l f(x\ \sin \frac{n\pi x}{l} dx.$$