

Use the method of separation of variables to show that the 1-D heat conduction problem:
find $u = u(x, y)$ so that

$$\frac{\partial u}{\partial t}(x, t) - \alpha \frac{\partial^2 u}{\partial x^2}(x, t) = 0, 0 < x < l, t > 0$$

$$u(0, t) = 0, t \geq 0$$

$$u(l, t) = 0, t \geq 0$$

$$u(x, 0) = f(x), 0 \leq x \leq l$$

has the solution

$$u(x, t) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{l} \exp \left[-\alpha \left(\frac{n\pi}{l} \right)^2 t \right]$$

where

$$c_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx.$$