Use the method of separation of variables to show that the 1-D heat conduction problem: find $u = u(x, y)$ so that

$$\frac{\partial u}{\partial t}(x,t) - a\ \frac{\partial^2 u}{\partial x^2}(x,t) = 0,\ 0 < x < l,\ t > 0$$
$$u(0,t) = 0,\ t \geq 0$$
$$u(l,t) = 0,\ t \geq 0$$
$$u(x,0) = f(x),\ 0 \leq x \leq l$$

has the solution

$$u(x,t) = \sum_{n=1}^{\infty} c_n \sin \frac{np\,x}{l} \exp\left[-a\left(\frac{np}{l}\right)^2 t \right]$$

where

$$c_n = \frac{2}{l}\int_0^l f(x) \sin \frac{np\,x}{l}\,dx.$$