HW Solutions

(48) Let Y = # packages returned (out of 3) want $P(Y=1)$

$Y \sim Bin(n=3, p=?)$ $p = P(\text{2 or more defective disks in a package})$

Let X count # defective out of 10 disks

$X \sim Bin(n=10, p=.01)$

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$
$$= 1 - (.99)^{10} - \binom{10}{1}(.01)(.99)^9$$
$$= .0043$$

$$P(Y=1) = \binom{3}{1}(.0043)(.9957)^2 =$$

(49) Yes, at least approximately binomial. We can think of chips being produced by a process which produces a defective w/ fixed probability 0.1 (this is a (reasonable) assumption)

(41) No ESP $\Longrightarrow X \sim Bin(n=10, p=1/2)$ where X counts correct guesses

$$P(X \geq 7) = \binom{10}{7}(1/2)^{10} + \binom{10}{8}(1/2)^{10} + \binom{10}{9}(1/2)^{10} + \binom{10}{10}(1/2)^{10}$$
$$= .1711$$

(54) Let Y count abandoned cars in one week. $Y \sim Poiss(\lambda = 2.2)$

(a) $P(Y=0) = \frac{(2.2)^0 e^{-2.2}}{0!} = .1108$

(b) $P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$
$$= 1 - e^{-2.2} - 2.2e^{-2.2}$$
$$= .6454$$

(55) Let X count # of errors in your article.

$$P(X=0) = P(X=0 \cap \text{typist A}) + P(X=0 \cap \text{typist B})$$
$$= P(\text{typist A})P(X=0|A) + P(\text{typist B})P(X=0|\text{typist B})$$
$$= 1/2 e^{-3} + 1/2 e^{-4.2}$$
$$= .0324$$

57) $Y \sim \text{Poisson}(\lambda = 3)$

(a) $P(Y \geq 3) = 1 - \left[e^{-3} + 3e^{-3} + \frac{3^2 e^{-3}}{2!}\right] = 0.5678$

(b) $P(Y \geq 3 \mid Y \geq 1) = \frac{P(Y \geq 3 \cap Y \geq 1)}{P(Y \geq 1)}$

$= \frac{P(Y \geq 3)}{P(Y \geq 1)}$

$= \frac{.5678}{1 - e^{-3}}$

58)

	Binomial probability	Poisson approximation
(a)	$\binom{8}{2}(.1)^2(.9)^6 = .1488$	$.8e^{-.8}/2 = .1438$
(b)	$\binom{10}{9}(.95)^9(.05) = .315$	$\frac{(9.5)^9 e^{-9.5}}{9!} = .1300$
(c)	$(.9)^{10} = .3487$	$e^{-1} = .3679$
(d)	$\binom{9}{4}(.2)^4(.8)^5 = .0661$	$\frac{(1.8)^4 e^{-1.8}}{4!} = .0723$

66) (a) $\left(\frac{26}{38}\right)^5 = .14995$ (b) $\left(\frac{26}{38}\right)^3\left(\frac{12}{38}\right) = .1012$

(67)

i	P(stronger team wins in i games)
4	$(.6)^4 = .1296$
5	$\binom{4}{3}(.4)^1(.6)^4 = .2074$
6	$\binom{5}{3}(.4)^2(.6)^4 = .2074$
7	$\binom{6}{3}(.4)^3(.6)^4 = .1659$

P(stronger team wins) $= .1296 + .2074 + .2074 + .1659 = .7103$

P(stronger wins 2-out-of-3) $= 2(.4)(.6)^2 + .6^2 = .648$

4-out-of-7 gives stronger team a better chance of winning the series.

(68) Let: y count # of games played

AAAA denote team A wins all 4; BBBB denote B wins all 4, etc...

y	$p(y)$
4	$P(AAAA) + P(BBBB) = 2(1/2)^4 = 1/8$
5	$2\binom{4}{3}(1/2)^1(1/2)^4 = 8(1/2)^5 = 1/4$
6	$2\binom{5}{3}(1/2)^2(1/2)^4 = 20(1/2)^6 = 5/16$
7	$2\binom{6}{3}(1/2)^3(1/2)^4 = 40(1/2)^7 = 5/16$

$E(Y) = (4)1/8 + 5(1/4) + 6(5/16) + 7(5/16) = 5.81$

(70) Let y count flips until 10 heads

$x = y - 10$ so $P(X=0) = P(Y=10)$; $P(X=1) = P(Y=11)$; and so on

$f(y) = \binom{y-1}{9}(1/2)^{10}(1/2)^{y-10} = \binom{y-1}{9}(1/2)^y \qquad y = 10, 11, \ldots$

~~f(x)=~~ Since $y = x + 10$

$f(x) = \binom{x+10-1}{9}(1/2)^{x+10} \qquad x = 0, 1, 2, \ldots$