Mthsc 860
Spring 2007
Homework Problem 2.8

Background:

Tridiagonal matrices

- arise in a variety of applications
- are often diagonally dominant or symmetric positive definite
- Heath displays their standard form and a GE algorithm when no pivoting is required (read Section 2.5.3 page 88).

Homework Problem 2.8. Write a Matlab program to

a) Generate an $n \times n$ tridiagonal matrix which is symmetric and positive definite by doing the following

i) specify n
ii) set B to be the lower triangular matrix with main diagonal and 1st lower diagonal stored as two vectors, **d** (main) and **e** (sub), (all other entries are zero). Let

$$d_j = 100/j, \ j = 1, \ \ldots, \ n$$

$$e_j = 100/(j-1), \ j = 2, \ \ldots, \ n$$

iii) Compute $A = BB^T$, storing A in vectors **a** and **b** as in the middle of page 88. (Note that storing **c** is unnecessary).

b) Find the Cholesky factorization of A, storing L in two vectors, **f** (main diagonal) and **g** (1st sub-diagonal). (Write your own program. Do not use the Matlab 'chol' routine.)

c) Compute $\|d-f\|_2$, $\|e-g\|_2$, $\dfrac{\|d-f\|_2}{\|d\|_2}$, $\dfrac{\|e-g\|_2}{\|e\|_2}$. (Type 'help norm' or 'doc norm' in Matlab)

Run your code for $n = 10, \ 100, \ 1000, \ 10000$. Include in your write-up the development of the factorization algorithm, the code, and results. No matrices or vectors need to be explicitly listed in the write-up.