Mthsc 860
Spring 2007
Homework Problem 5.7

a) Consider the nonlinear system

$$2x_1 - x_2 + \frac{1}{9}e^{-x_1} = -1 \quad (1)$$

$$-x_1 + 2x_2 + \frac{1}{9}e^{-x_2} = 1 \quad (2)$$

Write a code to solve the system using Newton's method. Take $x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

i) Determine the Newton's method iterate that satisfies a difference-between-successive-iterates tolerance of 10^{-10}.

ii) Using the results from (i) as the exact solution, rerun the code until a tolerance of 10^{-6} is met, and determine the rate of convergence r and the constant C in the rate formula.

b) Rewrite the system in (a) as

$$Kx + \phi(x) = b$$

where $K = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $\phi(x) = \begin{bmatrix} \frac{1}{9}e^{-x_1} \\ \frac{1}{9}e^{-x_2} \end{bmatrix}$, and $b = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

i) Do two iterations (by hand) of the fixed-point iteration

$$x^{(k+1)} = \frac{1}{2}(b - \phi(x^{(k)}) - Kx^{(k)} + 2x^{(k)})$$

for this system, using $x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

ii) Do two iterations (by hand) of the fixed-point iteration

$$x^{(k+1)} = K^{-1}(b - \phi(x^{(k)}))$$

for this system, using $x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

iii) which one do you think is going to converge faster?

c) Write a computer code to implement the fixed-point iterations outlined in part (b). For each case, use a tolerance of 10^{-6}, and with the results from (a)(i) as the exact solution, determine the rate of convergence r and the constant C in the rate formula. Also, compare the efficiency of these methods for this problem with your results for Newton's method.