Mthsc 860
Spring 2007
Homework Problem 8.6

a) Heath 8.5b

b) Heath 8.5c

c) Heath 8.5d

For all parts, use the Matlab routine **quad**. Compare results using tolerances of 1e-4 and 1e-8, using a single subroutine call (i.e. it is not necessary to consider the case where the integral is called separately in two appropriate intervals). Construct the plot (analogous to that in Figure 8.4) only for the case of using a tolerance of 1e-8. Here is an example program for Heath 8.5a:

```
Q = quad(@CP8_6_fcn_a,0,1,1e-3,1)

function f = CP8_6_fcn_a(x)
I1 = (0 <= x) & (x < 0.3);
I2 = (0.3 <= x) & (x < 1);
f = x; % Create y same size as x
f(I1) = 0*f(I1);
f(I2) = ones(size(f(I2)));
```

Normally the function defining the integrand would be of the simpler form

```
function f = CP8_6_fcn_a(x)
f = . . .;
```

The piecewise definition of the integrand requires some special consideration when using **quad**, as explained in the documentation at this website:

`http://www.mathworks.com/support/solutions/data/1-1A4J2.html?solution=1-1A4J2`

Here are some tips on how to make the plot similar to that in Figure 8.4. Note that the input parameter '1' in the last place in the statement 'Q = quad(...' above tells the code to print out more information (see the discussion of TRACE in the help documentation for **quad**). Copy the 4-column array of numbers output by **quad** and paste them into an array with these commands:

```
z = [
   (paste numbers here)
];
```

Then use this command (or your own variation on it) to make the plot.

```
plot(z(:,2),0,'+',z(:,2),CP8_6_fcn_a(z(:,2)),'o')
```

On finding the convergence rate for composite Newton-Cotes rules: We're looking for r for which the error in the composite rule behaves according to $error \approx Ch^r$. Given uniform spacings h_1 and h_2 and resulting errors in each case, E_1 and E_2, it's not difficult to show that r can be found using the equation

$$\ln \frac{E_2}{E_1} = r \ln \frac{h_2}{h_1}.$$

If a sequence of spacings $h_1,\ h_2,\ ,\ldots$ is generating a sequence of approximations with errors $E_1,\ E_2,\ ,\ldots,$ then one can calculate r at each new step using

$$\ln \frac{E_{i+1}}{E_i} = r \ln \frac{h_{i+1}}{h_i}.$$