

HW #1 (206)

10. (a) The distance from a point to the xy -plane is the absolute value of the z -coordinate of the point. Thus, the distance is $|-5| = 5$.

(b) Similarly, the distance to the yz -plane is the absolute value of the x -coordinate of the point: $|3| = 3$.

(c) The distance to the xz -plane is the absolute value of the y -coordinate of the point: $|7| = 7$.

(d) The point on the x -axis closest to $(3, 7, -5)$ is the point $(3, 0, 0)$. (Approach the x -axis perpendicularly.)

The distance from $(3, 7, -5)$ to the x -axis is the distance between these two points:

$$\sqrt{(3-3)^2 + (7-0)^2 + (-5-0)^2} = \sqrt{74} \approx 8.60.$$

(e) The point on the y -axis closest to $(3, 7, -5)$ is $(0, 7, 0)$. The distance between these points is

$$\sqrt{(3-0)^2 + (7-7)^2 + (-5-0)^2} = \sqrt{34} \approx 5.83.$$

(f) The point on the z -axis closest to $(3, 7, -5)$ is $(0, 0, -5)$. The distance between these points is

$$\sqrt{(3-0)^2 + (7-0)^2 + [-5-(-5)]^2} = \sqrt{58} \approx 7.62.$$

11. An equation of the sphere with center $(1, -4, 3)$ and radius 5 is $(x-1)^2 + [y-(-4)]^2 + (z-3)^2 = 5^2$ or

$(x-1)^2 + (y+4)^2 + (z-3)^2 = 25$. The intersection of this sphere with the xz -plane is the set of points on the sphere

whose y -coordinate is 0. Putting $y = 0$ into the equation, we have $(x-1)^2 + 4^2 + (z-3)^2 = 25$, $y = 0$ or

$(x-1)^2 + (z-3)^2 = 9$, $y = 0$, which represents a circle in the xz -plane with center $(1, 0, 3)$ and radius 3.

17. Completing squares in the equation $2x^2 - 8x + 2y^2 + 2z^2 + 24z = 1$ gives

$$2(x^2 - 4x + 4) + 2y^2 + 2(z^2 + 12z + 36) = 1 + 8 + 72 \Rightarrow 2(x-2)^2 + 2y^2 + 2(z+6)^2 = 81 \Rightarrow$$

$(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2}$, which we recognize as an equation of a sphere with center $(2, 0, -6)$ and radius

$$\sqrt{\frac{81}{2}} = 9/\sqrt{2}.$$

20. By Exercise 19(a), the midpoint of the diameter (and thus the center of the sphere) is $C(3, 2, 7)$. The radius is half the diameter, so $r = \frac{1}{2}\sqrt{(4-2)^2 + (3-1)^2 + (10-4)^2} = \frac{1}{2}\sqrt{44} = \sqrt{11}$. Therefore an equation of the sphere is

$$(x-3)^2 + (y-2)^2 + (z-7)^2 = 11.$$