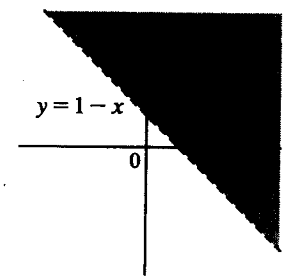
HW # 10

6. (a) $f(1,1) = \ln(1+1-1) = \ln 1 = 0$

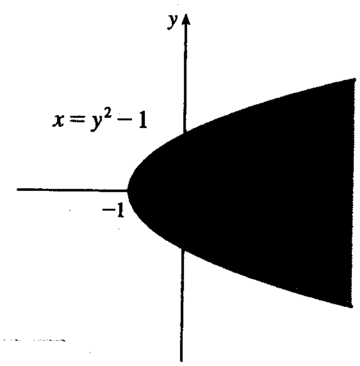
(b) $f(e,1) = \ln(e+1-1) = \ln e = 1$

(c) $\ln(x+y-1)$ is defined only when $x+y-1>0$, that is, $y > 1-x$. So the domain of f is $\{(x,y) \mid y > 1-x\}$.

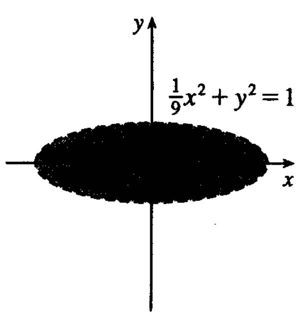
(d) Since $\ln(x+y-1)$ can be any real number, the range is $\mathbb{R}$.



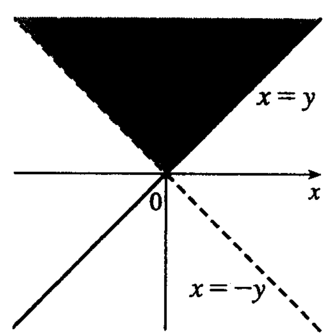
8. $\sqrt{1+x-y^2}$ is defined only when $1+x-y^2 \geq 0 \;\Rightarrow\; x \geq y^2-1$, so the domain of f is $\{(x,y) \mid x \geq y^2-1\}$, all those points on or to the right of the parabola $x = y^2-1$. The range of f is $[0,\infty)$.



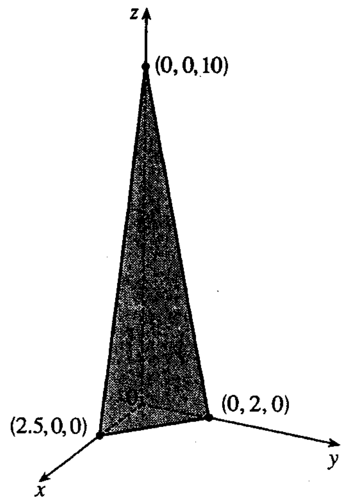
13. $\ln(9-x^2-9y^2)$ is defined only when $9-x^2-9y^2>0$, or $\frac{1}{9}x^2+y^2<1$. So the domain of f is $\left\{(x,y) \mid \frac{1}{9}x^2+y^2<1\right\}$, the interior of an ellipse.



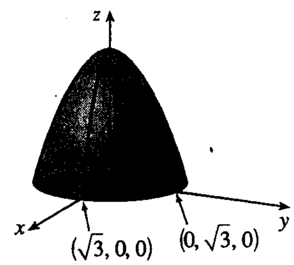
14. We need $y-x \geq 0$ or $y \geq x$ and $y+x>0$ or $x>-y$. Thus $D = \{(x,y) \mid -y < x \leq y, y>0\}$.



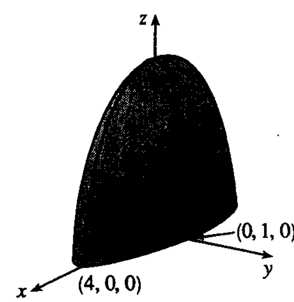
23. $z = 10 - 4x - 5y$ or $4x + 5y + z = 10$, a plane with intercepts 2.5, 2, and 10.



26. $z = 3 - x^2 - y^2$, a circular paraboloid with vertex at $(0, 0, 3)$.



28. $z = \sqrt{16 - x^2 - 16y^2}$ so $z \geq 0$ and $z^2 + x^2 + 16y^2 = 16$, the top half of an ellipsoid.



30. All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x, y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x, y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

(c) $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \dfrac{1}{1 + y^2}$, and in $y = 0$ is $z = \dfrac{1}{1 + x^2}$. In addition, we can see that f is close to 0 for large values of x and y, so this is graph I.

(d) $f(x, y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \quad \Rightarrow \quad y = \pm x$, so it must be graph IV.

(e) $f(x, y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \quad \Rightarrow \quad y = x$, so it must be graph II.

(f) $f(x, y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.