HW #11

5. $f(x,y) = 5x^3 - x^2y^2$ is a polynomial, and hence continuous, so $\lim_{(x,y)\to(1,2)} f(x,y) = f(1,2) = 5(1)^3 - (1)^2(2)^2 = 1$.

6. $-xy$ is a polynomial and therefore continuous. Since e^t is a continuous function, the composition e^{-xy} is also continuous. Similarly, $x + y$ is a polynomial and $\cos t$ is a continuous function, so the composition $\cos(x+y)$ is continuous. The product of continuous functions is continuous, so $f(x,y) = e^{-xy}\cos(x+y)$ is a continuous function and

$$\lim_{(x,y)\to(1,-1)} f(x,y) = f(1,-1) = e^{-(1)(-1)}\cos(1+(-1)) = e^1\cos 0 = e.$$

7. $f(x,y) = \dfrac{4-xy}{x^2+3y^2}$ is a rational function and hence continuous on its domain.

$(2,1)$ is in the domain of f, so f is continuous there and $\lim_{(x,y)\to(2,1)} f(x,y) = f(2,1) = \dfrac{4-(2)(1)}{(2)^2+3(1)^2} = \dfrac{2}{7}$.

12. $f(x,y) = 6x^3y/(2x^4+y^4)$. On the x-axis, $f(x,0) = 0$ for $x \neq 0$, so $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the x-axis. Approaching $(0,0)$ along the line $y = x$ gives $f(x,x) = 6x^4/(3x^4) = 2$ for $x \neq 0$, so along this line $f(x,y) \to 2$ as $(x,y) \to (0,0)$. Thus the limit does not exist.

18. $f(x,y) = xy^4/(x^2+y^8)$. On the x-axis, $f(x,0) = 0$ for $x \neq 0$, so $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along the x-axis. Approaching $(0,0)$ along the curve $x = y^4$ gives $f(y^4, y) = y^8/2y^8 = \frac{1}{2}$ for $y \neq 0$, so along this path $f(x,y) \to \frac{1}{2}$ as $(x,y) \to (0,0)$. Thus the limit does not exist.