HW #12.

15. $f(x,y) = y^5 - 3xy \;\Rightarrow\; f_x(x,y) = 0 - 3y = -3y$, $f_y(x,y) = 5y^4 - 3x$

17. $f(x,t) = e^{-t}\cos\pi x \;\Rightarrow\; f_x(x,t) = e^{-t}(-\sin\pi x)(\pi) = -\pi e^{-t}\sin\pi x$, $f_t(x,t) = e^{-t}(-1)\cos\pi x = -e^{-t}\cos\pi x$

19. $z = (2x+3y)^{10} \;\Rightarrow\; \dfrac{\partial z}{\partial x} = 10(2x+3y)^9 \cdot 2 = 20(2x+3y)^9$, $\dfrac{\partial z}{\partial y} = 10(2x+3y)^9 \cdot 3 = 30(2x+3y)^9$

39. $f(x,y) = \ln\left(x + \sqrt{x^2+y^2}\right) \;\Rightarrow$

$$f_x(x,y) = \frac{1}{x+\sqrt{x^2+y^2}}\left[1 + \tfrac{1}{2}(x^2+y^2)^{-1/2}(2x)\right] = \frac{1}{x+\sqrt{x^2+y^2}}\left(1 + \frac{x}{\sqrt{x^2+y^2}}\right),$$

so $f_x(3,4) = \dfrac{1}{3+\sqrt{3^2+4^2}}\left(1 + \dfrac{3}{\sqrt{3^2+4^2}}\right) = \frac{1}{8}\left(1+\frac{3}{5}\right) = \frac{1}{5}$.

41. $f(x,y,z) = \dfrac{y}{x+y+z} \;\Rightarrow\; f_y(x,y,z) = \dfrac{1(x+y+z) - y(1)}{(x+y+z)^2} = \dfrac{x+z}{(x+y+z)^2}$,

so $f_y(2,1,-1) = \dfrac{2+(-1)}{(2+1+(-1))^2} = \dfrac{1}{4}$.

45. $x^2+y^2+z^2 = 3xyz \;\Rightarrow\; \dfrac{\partial}{\partial x}(x^2+y^2+z^2) = \dfrac{\partial}{\partial x}(3xyz) \;\Rightarrow\; 2x + 0 + 2z\dfrac{\partial z}{\partial x} = 3y\left(x\dfrac{\partial z}{\partial x} + z\cdot 1\right) \;\Leftrightarrow$

$2z\dfrac{\partial z}{\partial x} - 3xy\dfrac{\partial z}{\partial x} = 3yz - 2x \;\Leftrightarrow\; (2z - 3xy)\dfrac{\partial z}{\partial x} = 3yz - 2x$, so $\dfrac{\partial z}{\partial x} = \dfrac{3yz-2x}{2z-3xy}$.

$\dfrac{\partial}{\partial y}(x^2+y^2+z^2) = \dfrac{\partial}{\partial y}(3xyz) \;\Rightarrow\; 0 + 2y + 2z\dfrac{\partial z}{\partial y} = 3x\left(y\dfrac{\partial z}{\partial y} + z\cdot 1\right) \;\Leftrightarrow\; 2z\dfrac{\partial z}{\partial y} - 3xy\dfrac{\partial z}{\partial y} = 3xz - 2y \;\Leftrightarrow$

$(2z-3xy)\dfrac{\partial z}{\partial y} = 3xz - 2y$, so $\dfrac{\partial z}{\partial y} = \dfrac{3xz-2y}{2z-3xy}$.

46. $yz = \ln(x+z) \;\Rightarrow\; \dfrac{\partial}{\partial x}(yz) = \dfrac{\partial}{\partial x}(\ln(x+z)) \;\Rightarrow\; y\dfrac{\partial z}{\partial x} = \dfrac{1}{x+z}\left(1 + \dfrac{\partial z}{\partial x}\right) \;\Leftrightarrow\; \left(y - \dfrac{1}{x+z}\right)\dfrac{\partial z}{\partial x} = \dfrac{1}{x+z}$,

so $\dfrac{\partial z}{\partial x} = \dfrac{1/(x+z)}{y - 1/(x+z)} = \dfrac{1}{y(x+z)-1}$.

$\dfrac{\partial}{\partial y}(yz) = \dfrac{\partial}{\partial y}(\ln(x+z)) \;\Rightarrow\; y\dfrac{\partial z}{\partial y} + z\cdot 1 = \dfrac{1}{x+z}\left(0 + \dfrac{\partial z}{\partial y}\right) \;\Leftrightarrow\; \left(y - \dfrac{1}{x+z}\right)\dfrac{\partial z}{\partial y} = -z$,

so $\dfrac{\partial z}{\partial y} = \dfrac{-z}{y - 1/(x+z)} = \dfrac{z(x+z)}{1 - y(x+z)}$.

51. $f(x,y) = x^3y^5 + 2x^4y \quad \Rightarrow \quad f_x(x,y) = 3x^2y^5 + 8x^3y$, $f_y(x,y) = 5x^3y^4 + 2x^4$. Then $f_{xx}(x,y) = 6xy^5 + 24x^2y$, $f_{xy}(x,y) = 15x^2y^4 + 8x^3$, $f_{yx}(x,y) = 15x^2y^4 + 8x^3$, and $f_{yy}(x,y) = 20x^3y^3$.

52. $f(x,y) = \sin^2(mx + ny) \quad \Rightarrow \quad f_x(x,y) = 2\sin(mx+ny)\cos(mx+ny)\cdot m = m\sin(2mx+2ny)$ [using the identity $\sin 2\theta = 2\sin\theta\cos\theta$], $f_y(x,y) = 2\sin(mx+ny)\cos(mx+ny)\cdot n = n\sin(2mx+2ny)$.

Then $f_{xx}(x,y) = m\cos(2mx+2ny)\cdot 2m = 2m^2\cos(2mx+2ny)$,

$f_{xy}(x,y) = m\cos(2mx+2ny)\cdot 2n = 2mn\cos(2mx+2ny)$,

$f_{yx}(x,y) = n\cos(2mx+2ny)\cdot 2m = 2mn\cos(2mx+2ny)$, and

$f_{yy}(x,y) = n\cos(2mx+2ny)\cdot 2n = 2n^2\cos(2mx+2ny)$.