

HW # 13

1. $z = f(x, y) = 4x^2 - y^2 + 2y \Rightarrow f_x(x, y) = 8x, f_y(x, y) = -2y + 2$, so $f_x(-1, 2) = -8, f_y(-1, 2) = -2$.

By Equation 2, an equation of the tangent plane is $z - 4 = f_x(-1, 2)[x - (-1)] + f_y(-1, 2)(y - 2) \Rightarrow z - 4 = -8(x + 1) - 2(y - 2)$ or $z = -8x - 2y$.

2. $z = f(x, y) = 3(x - 1)^2 + 2(y + 3)^2 + 7 \Rightarrow f_x(x, y) = 6(x - 1), f_y(x, y) = 4(y + 3)$, so $f_x(2, -2) = 6$ and $f_y(2, -2) = 4$. By Equation 2, an equation of the tangent plane is $z - 12 = f_x(2, -2)(x - 2) + f_y(2, -2)[y - (-2)] \Rightarrow z - 12 = 6(x - 2) + 4(y + 2)$ or $z = 6x + 4y + 8$.

3. $z = f(x, y) = \sqrt{xy} \Rightarrow f_x(x, y) = \frac{1}{2}(xy)^{-1/2} \cdot y = \frac{1}{2}\sqrt{y/x}, f_y(x, y) = \frac{1}{2}(xy)^{-1/2} \cdot x = \frac{1}{2}\sqrt{x/y}$, so $f_x(1, 1) = \frac{1}{2}$ and $f_y(1, 1) = \frac{1}{2}$. Thus an equation of the tangent plane is $z - 1 = f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) \Rightarrow z - 1 = \frac{1}{2}(x - 1) + \frac{1}{2}(y - 1)$ or $x + y - 2z = 0$.

4. $z = f(x, y) = y \ln x \Rightarrow f_x(x, y) = y/x, f_y(x, y) = \ln x$, so $f_x(1, 4) = 4, f_y(1, 4) = 0$, and an equation of the tangent plane is $z - 0 = f_x(1, 4)(x - 1) + f_y(1, 4)(y - 4) \Rightarrow z = 4(x - 1) + 0(y - 4)$ or $z = 4x - 4$.

5. $z = f(x, y) = y \cos(x - y) \Rightarrow f_x = y(-\sin(x - y)(1)) = -y \sin(x - y), f_y = y(-\sin(x - y)(-1)) + \cos(x - y) = y \sin(x - y) + \cos(x - y)$, so $f_x(2, 2) = -2 \sin(0) = 0, f_y(2, 2) = 2 \sin(0) + \cos(0) = 1$ and an equation of the tangent plane is $z - 2 = 0(x - 2) + 1(y - 2)$ or $z = y$.

6. $z = f(x, y) = e^{x^2 - y^2} \Rightarrow f_x(x, y) = 2xe^{x^2 - y^2}, f_y(x, y) = -2ye^{x^2 - y^2}$, so $f_x(1, -1) = 2, f_y(1, -1) = 2$. By Equation 2, an equation of the tangent plane is $z - 1 = f_x(1, -1)(x - 1) + f_y(1, -1)(y - (-1)) \Rightarrow z - 1 = 2(x - 1) + 2(y + 1)$ or $z = 2x + 2y + 1$.

11. $f(x, y) = x\sqrt{y}$. The partial derivatives are $f_x(x, y) = \sqrt{y}$ and $f_y(x, y) = \frac{x}{2\sqrt{y}}$, so $f_x(1, 4) = 2$ and $f_y(1, 4) = \frac{1}{4}$. Both f_x and f_y are continuous functions for $y > 0$, so by Theorem 8, f is differentiable at $(1, 4)$. By Equation 3, the linearization of f at $(1, 4)$ is given by $L(x, y) = f(1, 4) + f_x(1, 4)(x - 1) + f_y(1, 4)(y - 4) = 2 + 2(x - 1) + \frac{1}{4}(y - 4) = 2x + \frac{1}{4}y - 1$.

12. $f(x, y) = x^3y^4$. The partial derivatives are $f_x(x, y) = 3x^2y^4$ and $f_y(x, y) = 4x^3y^3$, so $f_x(1, 1) = 3$ and $f_y(1, 1) = 4$. Both f_x and f_y are continuous functions, so f is differentiable at $(1, 1)$ by Theorem 8. The linearization of f at $(1, 1)$ is given by $L(x, y) = f(1, 1) + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) = 1 + 3(x - 1) + 4(y - 1) = 3x + 4y - 6$.

16. $f(x, y) = \sin(2x + 3y)$. The partial derivatives are $f_x(x, y) = 2 \cos(2x + 3y)$ and $f_y(x, y) = 3 \cos(2x + 3y)$, so $f_x(-3, 2) = 2$ and $f_y(-3, 2) = 3$. Both f_x and f_y are continuous functions, so f is differentiable at $(-3, 2)$, and the linearization of f at $(-3, 2)$ is $L(x, y) = f(-3, 2) + f_x(-3, 2)(x + 3) + f_y(-3, 2)(y - 2) = 0 + 2(x + 3) + 3(y - 2) = 2x + 3y$.

25. $z = x^3 \ln(y^2) \Rightarrow dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = 3x^2 \ln(y^2) dx + x^3 \cdot \frac{1}{y^2}(2y) dy = 3x^2 \ln(y^2) dx + \frac{2x^3}{y} dy$

26. $v = y \cos xy \Rightarrow$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy = y(-\sin xy)y dx + [y(-\sin xy)x + \cos xy] dy = -y^2 \sin xy dx + (\cos xy - xy \sin xy) dy$$

31. $dx = \Delta x = 0.05, dy = \Delta y = 0.1, z = 5x^2 + y^2, z_x = 10x, z_y = 2y$. Thus when $x = 1$ and $y = 2$,

$$dz = z_x(1, 2) dx + z_y(1, 2) dy = (10)(0.05) + (4)(0.1) = 0.9 \text{ while}$$

$$\Delta z = f(1.05, 2.1) - f(1, 2) = 5(1.05)^2 + (2.1)^2 - 5 - 4 = 0.9225.$$