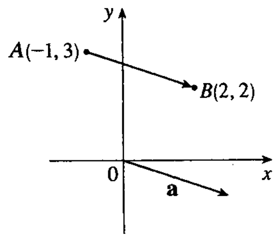
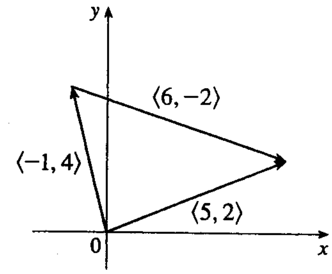
HW #2 (2.6)

9. $\mathbf{a} = \langle 2-(-1), 2-3 \rangle = \langle 3, -1 \rangle$



13. $\langle -1, 4 \rangle + \langle 6, -2 \rangle = \langle -1+6, 4+(-2) \rangle = \langle 5, 2 \rangle$



17. $\mathbf{a}+\mathbf{b} = \langle 5+(-3), -12+(-6) \rangle = \langle 2, -18 \rangle$

$2\mathbf{a}+3\mathbf{b} = \langle 10, -24 \rangle + \langle -9, -18 \rangle = \langle 1, -42 \rangle$

$|\mathbf{a}| = \sqrt{5^2+(-12)^2} = \sqrt{169} = 13$

$|\mathbf{a}-\mathbf{b}| = |\langle 5-(-3), -12-(-6) \rangle| = |\langle 8, -6 \rangle| = \sqrt{8^2+(-6)^2} = \sqrt{100} = 10$

23. The vector $8\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ has length $|8\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}| = \sqrt{8^2+(-1)^2+4^2} = \sqrt{81} = 9$, so by Equation 4 the unit vector with the same direction is $\frac{1}{9}(8\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) = \frac{8}{9}\,\mathbf{i} - \frac{1}{9}\,\mathbf{j} + \frac{4}{9}\,\mathbf{k}$.

24. $|\langle -2, 4, 2 \rangle| = \sqrt{(-2)^2+4^2+2^2} = \sqrt{24} = 2\sqrt{6}$, so a unit vector in the direction of $\langle -2, 4, 2 \rangle$ is $\mathbf{u} = \dfrac{1}{2\sqrt{6}}\langle -2, 4, 2 \rangle$.

A vector in the same direction but with length 6 is $6\mathbf{u} = 6 \cdot \dfrac{1}{2\sqrt{6}}\langle -2, 4, 2 \rangle = \left\langle -\dfrac{6}{\sqrt{6}}, \dfrac{12}{\sqrt{6}}, \dfrac{6}{\sqrt{6}} \right\rangle$ or $\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6} \rangle$.

32. Call the two tensile forces $\mathbf{T}_3$ and $\mathbf{T}_5$, corresponding to the ropes of length 3 m and 5 m. In terms of vertical and horizontal components,

$$\mathbf{T}_3 = -|\mathbf{T}_3|\cos 52^\circ\mathbf{i} + |\mathbf{T}_3|\sin 52^\circ\mathbf{j} \quad \textbf{(1)} \qquad \text{and} \qquad \mathbf{T}_5 = |\mathbf{T}_5|\cos 40^\circ\mathbf{i} + |\mathbf{T}_5|\sin 40^\circ\mathbf{j} \quad \textbf{(2)}$$

The resultant of these forces, $\mathbf{T}_3 + \mathbf{T}_5$, counterbalances the force of gravity acting on the decoration [which is $-5g\,\mathbf{j} \approx -5(9.8)\,\mathbf{j} = -49\,\mathbf{j}$]. So $\mathbf{T}_3 + \mathbf{T}_5 = 49\,\mathbf{j}$. Hence

$\mathbf{T}_3 + \mathbf{T}_5 = (-|\mathbf{T}_3|\cos 52^\circ + |\mathbf{T}_5|\cos 40^\circ)\,\mathbf{i} + (|\mathbf{T}_3|\sin 52^\circ + |\mathbf{T}_5|\sin 40^\circ)\,\mathbf{j} = 49\,\mathbf{j}$.

Thus $-|\mathbf{T}_3|\cos 52^\circ + |\mathbf{T}_5|\cos 40^\circ = 0$ and $|\mathbf{T}_3|\sin 52^\circ + |\mathbf{T}_5|\sin 40^\circ = 49$.

From the first of these two equations $|\mathbf{T}_3| = |\mathbf{T}_5|\dfrac{\cos 40^\circ}{\cos 52^\circ}$. Substituting this into the second equation gives

$|\mathbf{T}_5| = \dfrac{49}{\cos 40^\circ \tan 52^\circ + \sin 40^\circ} \approx 30$ N. Therefore, $|\mathbf{T}_3| = |\mathbf{T}_5|\dfrac{\cos 40^\circ}{\cos 52^\circ} \approx 38$ N. Finally, from **(1)** and **(2)**,

$\mathbf{T}_3 \approx -23\,\mathbf{i} + 30\,\mathbf{j}$, and $\mathbf{T}_5 \approx 23\,\mathbf{i} + 19\,\mathbf{j}$.