HW 3

9. $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta = (6)(5)\cos\frac{2\pi}{3} = 30\left(-\frac{1}{2}\right) = -15$

11. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all unit vectors, so the triangle is an equilateral triangle. Thus the angle between $\mathbf{u}$ and $\mathbf{v}$ is $60°$ and $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}|\,|\mathbf{v}|\cos 60° = (1)(1)\left(\frac{1}{2}\right) = \frac{1}{2}$. If $\mathbf{w}$ is moved so it has the same initial point as $\mathbf{u}$, we can see that the angle between them is $120°$ and we have $\mathbf{u}\cdot\mathbf{w} = |\mathbf{u}|\,|\mathbf{w}|\cos 120° = (1)(1)\left(-\frac{1}{2}\right) = -\frac{1}{2}$.

15. $|\mathbf{a}| = \sqrt{(-8)^2+6^2} = 10$, $|\mathbf{b}| = \sqrt{\left(\sqrt{7}\right)^2+3^2} = 4$, and $\mathbf{a}\cdot\mathbf{b} = (-8)\left(\sqrt{7}\right)+(6)(3) = 18-8\sqrt{7}$. From Corollary 6, we have $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{18-8\sqrt{7}}{10\cdot 4} = \dfrac{9-4\sqrt{7}}{20}$. So the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = \cos^{-1}\left(\dfrac{9-4\sqrt{7}}{20}\right) \approx 95°$.

19. $|\mathbf{a}| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$, $|\mathbf{b}| = \sqrt{1^2+2^2+(-3)^2} = \sqrt{14}$, and $\mathbf{a}\cdot\mathbf{b} = (0)(1)+(1)(2)+(1)(-3) = -1$. Then $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{-1}{\sqrt{2}\cdot\sqrt{14}} = \dfrac{-1}{2\sqrt{7}}$ and $\theta = \cos^{-1}\left(-\dfrac{1}{2\sqrt{7}}\right) \approx 101°$.

23. (a) $\mathbf{a}\cdot\mathbf{b} = (-5)(6)+(3)(-8)+(7)(2) = -40 \neq 0$, so $\mathbf{a}$ and $\mathbf{b}$ are not orthogonal. Also, since $\mathbf{a}$ is not a scalar multiple of $\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are not parallel.

(b) $\mathbf{a}\cdot\mathbf{b} = (4)(-3)+(6)(2) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

(c) $\mathbf{a}\cdot\mathbf{b} = (-1)(3)+(2)(4)+(5)(-1) = 0$, so $\mathbf{a}$ and $\mathbf{b}$ are orthogonal (and not parallel).

(d) Because $\mathbf{a} = -\frac{2}{3}\,\mathbf{b}$, $\mathbf{a}$ and $\mathbf{b}$ are parallel.

29. Since $|\langle 3,4,5\rangle| = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$, using Equations 8 and 9 we have $\cos\alpha = \frac{3}{5\sqrt{2}}$, $\cos\beta = \frac{4}{5\sqrt{2}}$, and $\cos\gamma = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$. The direction angles are given by $\alpha = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right) \approx 65°$, $\beta = \cos^{-1}\left(\frac{4}{5\sqrt{2}}\right) \approx 56°$, and $\gamma = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$.

34. Since $\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1$, $\cos^2\gamma = 1-\cos^2\alpha-\cos^2\beta = 1-\cos^2\left(\frac{\pi}{4}\right)-\cos^2\left(\frac{\pi}{3}\right) = 1-\left(\frac{1}{\sqrt{2}}\right)^2-\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Thus $\cos\gamma = \pm\frac{1}{2}$ and $\gamma = \frac{\pi}{3}$ or $\gamma = \frac{2\pi}{3}$.

35. $|\mathbf{a}| = \sqrt{3^2+(-4)^2} = 5$. The scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{3\cdot 5+(-4)\cdot 0}{5} = 3$ and the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\dfrac{\mathbf{a}}{|\mathbf{a}|} = 3\cdot\frac{1}{5}\langle 3,-4\rangle = \left\langle\frac{9}{5},-\frac{12}{5}\right\rangle$.

39. $|\mathbf{a}| = \sqrt{4+1+16} = \sqrt{21}$ so the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{0-1+2}{\sqrt{21}} = \dfrac{1}{\sqrt{21}}$ while the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \dfrac{1}{\sqrt{21}}\dfrac{\mathbf{a}}{|\mathbf{a}|} = \dfrac{1}{\sqrt{21}}\cdot\dfrac{2\,\mathbf{i}-\mathbf{j}+4\,\mathbf{k}}{\sqrt{21}} = \frac{1}{21}(2\,\mathbf{i}-\mathbf{j}+4\,\mathbf{k}) = \frac{2}{21}\,\mathbf{i}-\frac{1}{21}\,\mathbf{j}+\frac{4}{21}\,\mathbf{k}$.

45. The displacement vector is $\mathbf{D} = (6-0)\,\mathbf{i}+(12-10)\,\mathbf{j}+(20-8)\,\mathbf{k} = 6\,\mathbf{i}+2\,\mathbf{j}+12\,\mathbf{k}$ so by Equation 12 the work done is $W = \mathbf{F}\cdot\mathbf{D} = (8\,\mathbf{i}-6\,\mathbf{j}+9\,\mathbf{k})\cdot(6\,\mathbf{i}+2\,\mathbf{j}+12\,\mathbf{k}) = 48-12+108 = 144$ joules.