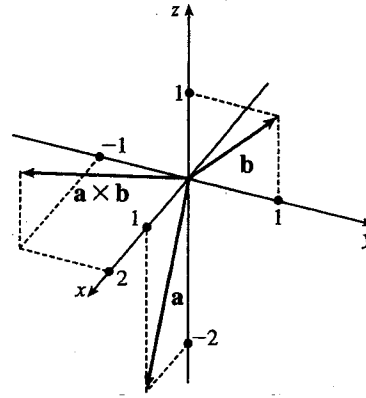


HW 4

$$8. \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \mathbf{k}$$

$$= 2\mathbf{i} - \mathbf{j} + \mathbf{k}$$



14. Using Theorem 6, we have $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin \theta = (5)(10) \sin 60^\circ = 25\sqrt{3}$. By the right-hand rule, $\mathbf{u} \times \mathbf{v}$ is directed into the page.

$$17. \mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} \mathbf{k} = (6-1)\mathbf{i} - (3-0)\mathbf{j} + (1-0)\mathbf{k} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$$

$$\mathbf{b} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} \mathbf{k} = (1-6)\mathbf{i} - (0-3)\mathbf{j} + (0-1)\mathbf{k} = -5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

Notice $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ here, as we know is always true by Theorem 8.

19. We know that the cross product of two vectors is orthogonal to both. So we calculate

$$\langle 1, -1, 1 \rangle \times \langle 0, 4, 4 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 0 & 4 & 4 \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 4 & 4 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -1 \\ 0 & 4 \end{vmatrix} \mathbf{k} = -8\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}.$$

So two unit vectors orthogonal to both are $\pm \frac{\langle -8, -4, 4 \rangle}{\sqrt{64+16+16}} = \pm \frac{\langle -8, -4, 4 \rangle}{4\sqrt{6}}$, that is, $\langle -\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$

and $\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}} \rangle$.

$$34. \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = 1 \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = -2 - 2 + 0 = -4.$$

So the volume of the parallelepiped determined by \mathbf{a} , \mathbf{b} , and \mathbf{c} is $|-4| = 4$ cubic units.

39. The magnitude of the torque is $|\boldsymbol{\tau}| = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}| |\mathbf{F}| \sin \theta = (0.18 \text{ m})(60 \text{ N}) \sin(70 + 10)^\circ = 10.8 \sin 80^\circ \approx 10.6 \text{ N}\cdot\text{m}$.

40. $|\mathbf{r}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$ ft. A line drawn from the point P to the point of application of the force makes an angle of $180^\circ - (45 + 30)^\circ = 105^\circ$ with the force vector. Therefore,

$$|\boldsymbol{\tau}| = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}| |\mathbf{F}| \sin \theta = (4\sqrt{2})(36) \sin 105^\circ \approx 197 \text{ ft}\cdot\text{lb}.$$