HW # 7

3. $\lim_{t\to 0^+} \cos t = \cos 0 = 1$, $\lim_{t\to 0^+} \sin t = \sin 0 = 0$, $\lim_{t\to 0^+} t\ln t = \lim_{t\to 0^+} \frac{\ln t}{1/t} = \lim_{t\to 0^+} \frac{1/t}{-1/t^2} = \lim_{t\to 0^+} -t = 0$

[by l'Hospital's Rule]. Thus $\lim_{t\to 0^+} \langle \cos t, \sin t, t\ln t\rangle = \left\langle \lim_{t\to 0^+} \cos t, \lim_{t\to 0^+} \sin t, \lim_{t\to 0^+} t\ln t \right\rangle = \langle 1, 0, 0\rangle$.

4. $\lim_{t\to 0} \frac{e^t - 1}{t} = \lim_{t\to 0} \frac{e^t}{1} = 1$ [using l'Hospital's Rule],

$\lim_{t\to 0} \frac{\sqrt{1+t}-1}{t} = \lim_{t\to 0} \frac{\sqrt{1+t}-1}{t} \cdot \frac{\sqrt{1+t}+1}{\sqrt{1+t}+1} = \lim_{t\to 0} \frac{1}{\sqrt{1+t}+1} = \frac{1}{2}$, $\lim_{t\to 0} \frac{3}{1+t} = 3$.

Thus the given limit equals $\langle 1, \frac{1}{2}, 3\rangle$.

15. Taking $\mathbf{r}_0 = \langle 0,0,0\rangle$ and $\mathbf{r}_1 = \langle 1,2,3\rangle$, we have from Equation 13.5.4 [ET 12.5.4]

$\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)\,\langle 0,0,0\rangle + t\,\langle 1,2,3\rangle, 0 \le t \le 1$ or $\mathbf{r}(t) = \langle t, 2t, 3t\rangle, 0 \le t \le 1$.

Parametric equations are $x = t,\ y = 2t,\ z = 3t,\ 0 \le t \le 1$.

16. Taking $\mathbf{r}_0 = \langle 1,0,1\rangle$ and $\mathbf{r}_1 = \langle 2,3,1\rangle$, we have from Equation 13.5.4 [ET 12.5.4]

$\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)\,\langle 1,0,1\rangle + t\,\langle 2,3,1\rangle, 0 \le t \le 1$ or $\mathbf{r}(t) = \langle 1+t, 3t, 1\rangle, 0 \le t \le 1$.

Parametric equations are $x = 1+t,\ y = 3t,\ z = 1,\ 0 \le t \le 1$.