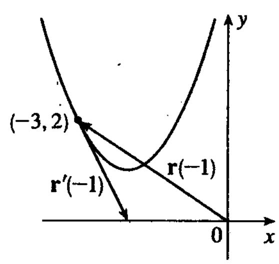
HW #8

3. Since $(x+2)^2 = t^2 = y - 1 \Rightarrow y = (x+2)^2 - 1$, the curve is a parabola.

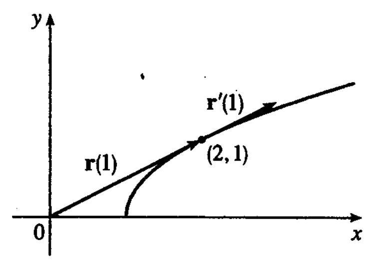
(a), (c)



(b) $\mathbf{r}'(t) = \langle 1, 2t \rangle$, $\mathbf{r}'(-1) = \langle 1, -2 \rangle$

4. Since $x = 1 + t = 1 + y^2$, the curve is part of a parabola. Here we have $y \geq 0$.

(a), (c)



(b) $\mathbf{r}'(t) = \left\langle 1, \dfrac{1}{2\sqrt{t}} \right\rangle$, $\mathbf{r}'(1) = \langle 1, \frac{1}{2} \rangle$

9. $\mathbf{r}'(t) = \left\langle \dfrac{d}{dt}[t \sin t], \dfrac{d}{dt}[t^2], \dfrac{d}{dt}[t \cos 2t] \right\rangle = \langle t\cos t + \sin t, 2t, t(-\sin 2t)\cdot 2 + \cos 2t \rangle$
$= \langle t\cos t + \sin t, 2t, \cos 2t - 2t \sin 2t \rangle$

10. $\mathbf{r}(t) = \langle \tan t, \sec t, 1/t^2 \rangle \Rightarrow \mathbf{r}'(t) = \langle \sec^2 t, \sec t \tan t, -2/t^3 \rangle$

14. $\mathbf{r}'(t) = [at(-3\sin 3t) + a\cos 3t]\,\mathbf{i} + b \cdot 3\sin^2 t \cos t\,\mathbf{j} + c \cdot 3\cos^2 t(-\sin t)\,\mathbf{k}$
$= (a\cos 3t - 3at\sin 3t)\,\mathbf{i} + 3b\sin^2 t\,\cos t\,\mathbf{j} - 3c\cos^2 t\,\sin t\,\mathbf{k}$

17. $\mathbf{r}'(t) = \langle -te^{-t} + e^{-t}, 2/(1+t^2), 2e^t \rangle \Rightarrow \mathbf{r}'(0) = \langle 1, 2, 2 \rangle$. So $|\mathbf{r}'(0)| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$ and
$\mathbf{T}(0) = \dfrac{\mathbf{r}'(0)}{|\mathbf{r}'(0)|} = \frac{1}{3}\langle 1, 2, 2 \rangle = \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$.

18. $\mathbf{r}'(t) = \dfrac{2}{\sqrt{t}}\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k} \Rightarrow \mathbf{r}'(1) = 2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$. Thus
$\mathbf{T}(1) = \dfrac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \dfrac{1}{\sqrt{2^2 + 2^2 + 1^2}}(2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}) = \frac{1}{3}(2\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}) = \frac{2}{3}\,\mathbf{i} + \frac{2}{3}\,\mathbf{j} + \frac{1}{3}\,\mathbf{k}$.

23. The vector equation for the curve is $\mathbf{r}(t) = \langle 1 + 2\sqrt{t}, t^3 - t, t^3 + t \rangle$, so $\mathbf{r}'(t) = \langle 1/\sqrt{t}, 3t^2 - 1, 3t^2 + 1 \rangle$. The point $(3, 0, 2)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle 1, 2, 4 \rangle$. Thus, the tangent line goes through the point $(3, 0, 2)$ and is parallel to the vector $\langle 1, 2, 4 \rangle$. Parametric equations are $x = 3 + t$, $y = 2t$, $z = 2 + 4t$.

24. The vector equation for the curve is $\mathbf{r}(t) = \left\langle e^t, te^t, te^{t^2} \right\rangle$, so $\mathbf{r}'(t) = \left\langle e^t, te^t + e^t, 2t^2 e^{t^2} + e^{t^2} \right\rangle$. The point $(1, 0, 0)$ corresponds to $t = 0$, so the tangent vector there is $\mathbf{r}'(0) = \langle 1, 1, 1 \rangle$. Thus, the tangent line is parallel to the vector $\langle 1, 1, 1 \rangle$ and includes the point $(1, 0, 0)$. Parametric equations are $x = 1 + 1 \cdot t = 1 + t$, $y = 0 + 1 \cdot t = t$, $z = 0 + 1 \cdot t = t$.

33. $$\int_0^1 (16t^3\,\mathbf{i} - 9t^2\,\mathbf{j} + 25t^4\,\mathbf{k})\,dt = \left(\int_0^1 16t^3\,dt\right)\mathbf{i} - \left(\int_0^1 9t^2\,dt\right)\mathbf{j} + \left(\int_0^1 25t^4\,dt\right)\mathbf{k}$$
$$= \left[4t^4\right]_0^1\,\mathbf{i} - \left[3t^3\right]_0^1\,\mathbf{j} + \left[5t^5\right]_0^1\,\mathbf{k} = 4\,\mathbf{i} - 3\,\mathbf{j} + 5\,\mathbf{k}$$

34. $$\int_0^1 \left(\frac{4}{1+t^2}\,\mathbf{j} + \frac{2t}{1+t^2}\,\mathbf{k}\right) dt = \left[4\tan^{-1}t\,\mathbf{j} + \ln(1+t^2)\,\mathbf{k}\right]_0^1 = \left[4\tan^{-1}1\,\mathbf{j} + \ln 2\,\mathbf{k}\right] - \left[4\tan^{-1}0\,\mathbf{j} + \ln 1\,\mathbf{k}\right]$$
$$= 4\left(\tfrac{\pi}{4}\right)\mathbf{j} + \ln 2\,\mathbf{k} - 0\,\mathbf{j} - 0\,\mathbf{k} = \pi\,\mathbf{j} + \ln 2\,\mathbf{k}$$

37. $$\int (e^t\,\mathbf{i} + 2t\,\mathbf{j} + \ln t\,\mathbf{k})\;dt = \left(\int e^t\,dt\right)\mathbf{i} + \left(\int 2t\,dt\right)\mathbf{j} + \left(\int \ln t\,dt\right)\mathbf{k}$$
$$= e^t\,\mathbf{i} + t^2\,\mathbf{j} + (t\ln t - t)\,\mathbf{k} + \mathbf{C},$$ where $\mathbf{C}$ is a vector constant of integration.