HW #9

1. $\mathbf{r}(t) = \langle 2\sin t, 5t, 2\cos t\rangle \;\Rightarrow\; \mathbf{r}'(t) = \langle 2\cos t, 5, -2\sin t\rangle \;\Rightarrow\; |\mathbf{r}'(t)| = \sqrt{(2\cos t)^2 + 5^2 + (-2\sin t)^2} = \sqrt{29}$.

Then using Formula 3, we have $L = \int_{-10}^{10} |\mathbf{r}'(t)|\,dt = \int_{-10}^{10} \sqrt{29}\,dt = \sqrt{29}\,t\Big]_{-10}^{10} = 20\sqrt{29}$.

2. $\mathbf{r}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3\right\rangle \;\Rightarrow\; \mathbf{r}'(t) = \left\langle 2, 2t, t^2\right\rangle \;\Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{2^2 + (2t)^2 + (t^2)^2} = \sqrt{4 + 4t^2 + t^4} = \sqrt{(2+t^2)^2} = 2 + t^2$ for $0 \le t \le 1$. Then using Formula 3, we have

$L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 (2+t^2)\,dt = 2t + \frac{1}{3}t^3\Big]_0^1 = \frac{7}{3}$.

3. $\mathbf{r}(t) = \sqrt{2}\,t\,\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k} \;\Rightarrow\; \mathbf{r}'(t) = \sqrt{2}\,\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k} \;\Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{\left(\sqrt{2}\right)^2 + (e^t)^2 + (-e^{-t})^2} = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$ [since $e^t + e^{-t} > 0$].

Then $L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 (e^t + e^{-t})\,dt = \left[e^t - e^{-t}\right]_0^1 = e - e^{-1}$.

4. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \ln\cos t\,\mathbf{k} \;\Rightarrow\; \mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \dfrac{-\sin t}{\cos t}\,\mathbf{k} = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} - \tan t\,\mathbf{k}$,

$|\mathbf{r}'(t)| = \sqrt{(-\sin t)^2 + \cos^2 t + (-\tan t)^2} = \sqrt{1 + \tan^2 t} = \sqrt{\sec^2 t} = |\sec t|$. Since $\sec t > 0$ for $0 \le t \le \pi/4$, here we can say $|\mathbf{r}'(t)| = \sec t$. Then

$$L = \int_0^{\pi/4} \sec t\,dt = \Big[\ln|\sec t + \tan t|\Big]_0^{\pi/4} = \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$
$$= \ln\left|\sqrt{2} + 1\right| - \ln|1 + 0| = \ln(\sqrt{2} + 1).$$

5. $\mathbf{r}(t) = \mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k} \;\Rightarrow\; \mathbf{r}'(t) = 2t\,\mathbf{j} + 3t^2\,\mathbf{k} \;\Rightarrow\; |\mathbf{r}'(t)| = \sqrt{4t^2 + 9t^4} = t\sqrt{4 + 9t^2}$ [since $t \ge 0$].

Then $L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 t\sqrt{4 + 9t^2}\,dt = \frac{1}{18}\cdot\frac{2}{3}(4 + 9t^2)^{3/2}\Big]_0^1 = \frac{1}{27}(13^{3/2} - 4^{3/2}) = \frac{1}{27}(13^{3/2} - 8)$.

6. $\mathbf{r}(t) = 12t\,\mathbf{i} + 8t^{3/2}\,\mathbf{j} + 3t^2\,\mathbf{k} \;\Rightarrow\; \mathbf{r}'(t) = 12\,\mathbf{i} + 12\sqrt{t}\,\mathbf{j} + 6t\,\mathbf{k} \;\Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{144 + 144t + 36t^2} = \sqrt{36(t+2)^2} = 6\,|t+2| = 6(t+2)$ for $0 \le t \le 1$. Then

$L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 6(t+2)\,dt = \Big[3t^2 + 12t\Big]_0^1 = 15$.