

2.27 The sample space is $\{SS, SR, SL, RS, RR, RL, LS, LR, LL\}$. Let A be the event of at least one vehicle turning left and B be the event that at least one vehicle turns. Assuming equally likely outcomes, we have

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} = \frac{P(SL \cup RL \cup LS \cup LR \cup LL)}{1 - P(SS)} = \frac{5/9}{8/9} = \frac{5}{8}$$

2.34 a. $P(A \cup B) \leq 1$

$$\text{i.e., } P(A) + P(B) - P(AB) \leq 1$$

$$\text{i.e., } P(AB) \geq P(A) + P(B) - 1$$

b. $P(\text{exactly one of the events occurs})$

$$= P(A\bar{B} \cup \bar{A}B) = P(A\bar{B}) + P(\bar{A}B)$$

$$= [P(A) - P(AB)] + [P(B) - P(AB)] = P(A) + P(B) - 2P(AB)$$

2.35 a. Let $A_i =$ event that i^{th} resistor has resistance between 9.5 and 10.5 ohms, $i = 1, 2$. Then

$$P(A_i) = 1 - 0.05 - 0.10 = 0.85, \text{ and}$$

$$P(\text{both have actual values between 9.5 and 10.5})$$

$$= P(A_1 A_2) = P(A_1) P(A_2) = (0.85)(0.85) = 0.7225.$$

b. Let $E_i =$ event that i^{th} resistor has resistance in excess of 10.5 ohms, $i = 1, 2$. Then

$$P(\text{at least one has an actual value greater than 10.5})$$

$$= P(E_1 E_2 \cup E_1 \bar{E}_2 \cup \bar{E}_1 E_2) = P(E_1) P(E_2) + P(E_1) P(\bar{E}_2) + P(\bar{E}_1) P(E_2)$$

$$= (0.1)(0.1) + 2(0.1)(0.9) = 0.19 \text{ or}$$

$$P(\text{at least one has an actual value greater than 10.5})$$

$$= 1 - P(\bar{E}_1 \bar{E}_2) = 1 - (0.9)^2 = 0.19.$$

2.37 Let $C_i =$ event that relay i closes properly, $i = 1, 2$. Then

$$P(\text{current flows in series system}) = P(\text{both relays are closed})$$

$$= P(C_1 C_2) = P(C_1) P(C_2) = (0.9)(0.9) = 0.81$$

$$P(\text{current flows in parallel system}) = P(\text{at least one of the relays is closed})$$

$$= P(C_1 \cup C_2) = P(C_1) + P(C_2) - P(C_1 C_2)$$

$$= 0.9 + 0.9 - (0.9)(0.9) = 0.99$$

2.38 Let D = defective, and L_i = the line from which the motor came, $i = 1, 2$.

$$\begin{aligned}P(L_1|D) &= \frac{P(L_1)P(D|L_1)}{P(D|L_1)P(L_1) + P(D|L_2)P(L_2)} \\ &= \frac{(0.5)(0.10)}{(0.10)(0.5) + (0.15)(0.5)} = 0.40\end{aligned}$$

2.39 Let F denote the event that a worker fails to learn the skill correctly.

$$\begin{aligned}P(A|F) &= \frac{P(A)P(F|A)}{P(F|A)P(A) + P(F|B)P(B)} \\ &= \frac{(0.70)(0.20)}{(0.20)(0.70) + (0.10)(0.30)} = 0.8235\end{aligned}$$

2.43 $P(A)P(B|A)P(C|AB) = P(A) \cdot \frac{P(AB)}{P(A)} \cdot \frac{P(ABC)}{P(AB)} = P(ABC)$