HW #3 MMSC 400

3.1 $P(X=0) = P(\text{3 males chosen})$

$$= \frac{\begin{pmatrix}\text{number of ways of}\\ \text{choosing 3 out of 4}\end{pmatrix}\begin{pmatrix}\text{number of ways of}\\ \text{choosing 0 out of 6}\end{pmatrix}}{\begin{pmatrix}\text{total number of}\\ \text{ways of choosing 3 out of 10}\end{pmatrix}}$$

$$= \frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = \frac{1}{30}$$

$$P(X=1) = P(\text{2 males and 1 female chosen}) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = \frac{3}{10}$$

$$P(X=2) = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{1}{2} \qquad P(X=3) = \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \frac{1}{6}$$

3.2 **a.** $P(X=0) = \begin{pmatrix}\text{number of ways of}\\ \text{choosing 0 out of 4}\end{pmatrix}\begin{pmatrix}\text{no chosen household head}\\ \text{has income } <\$18,000\end{pmatrix}$

$$= \binom{4}{0}\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$P(X=1) = \begin{pmatrix}\text{number of ways of}\\ \text{choosing 1 out of 4}\end{pmatrix}\begin{pmatrix}\text{only one given household head of}\\ \text{those chosen has income } <\$18,000\end{pmatrix}$

$$= \binom{4}{1}\left(\frac{1}{2}\right)^4 = \frac{1}{4}$$

$$P(X=2) = \binom{4}{2}\left(\frac{1}{2}\right)^4 = \frac{3}{8}$$

$$P(X=3) = \binom{4}{3}\left(\frac{1}{2}\right)^4 = \frac{1}{4} \qquad P(X=4) = \binom{4}{4}\left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

b. $P(X=0) = \frac{1}{16}$; i.e., it is unlikely to see all four below \$18,000 in this poll.

3.4 Assume that, for a given person, the outcomes were equally likely. Then

$$P(X=0) = \binom{3}{0}\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$P(X=1) = \binom{3}{1}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$$

$$P(X=2) = \binom{3}{2}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$$

$$P(X=3) = \binom{3}{3}\left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

3.9 **a.** $$p(x) = \frac{\left(\begin{array}{c}\text{number of ways of}\\ \text{choosing } x \text{ from } 2\end{array}\right)\left(\begin{array}{c}\text{number of ways of}\\ \text{choosing } 2-x \text{ from } 2\end{array}\right)}{\left(\begin{array}{c}\text{total number of ways of}\\ \text{choosing a sample of 2 from 4}\end{array}\right)}$$

$$\frac{\binom{2}{x}\binom{2}{2-x}}{\binom{4}{2}} \quad x = 0, 1, 2;$$

i.e,

x	$p(x)$
0	1/6
1	2/3
2	1/6

b. $$p(x) = \frac{\binom{1}{x}\binom{3}{2-x}}{\binom{4}{2}} \quad x = 0, 1;$$

i.e.,

x	$p(x)$
0	1/2
1	1/2

c. $P(X=0) = 1$

3.11 Let X = number on the ticket drawn, and G_i = net gain for box i, i = I, II. Then $G_i = X - 1$.

a. $$E(G_I) = \sum_{x=0}^{2} (x-1)p_I(x) = (-1)\left(\frac{1}{3}\right)+0\left(\frac{1}{3}\right)+1\left(\frac{1}{3}\right) = 0$$

$$E(G_I^2) = \sum_{x=0}^{2} (x-1)^2 p_I(x) = 1\left(\frac{1}{3}\right)+0\left(\frac{1}{3}\right)+1\left(\frac{1}{3}\right) = \frac{2}{3}$$

$$V(G_I) = E(G_I^2) - [E(G_I)]^2 = \frac{2}{3} - 0 = \frac{2}{3}$$

b. $$E(G_{II}) = \sum_{x=0}^{2} (x-1)p_{II}(x) = (-1)\left(\frac{3}{5}\right)+0\left(\frac{1}{5}\right)+3\left(\frac{1}{5}\right) = 0$$

$$E(G_{II}^2) = \sum_{x=0}^{2} (x-1)^2 p_{II}(x) = 1\left(\frac{3}{5}\right)+0\left(\frac{1}{5}\right)+9\left(\frac{1}{5}\right) = \frac{12}{5}$$

$$V(G_{II}) = E(G_{II}^2) - [E(G_{II})]^2 = \frac{12}{5} - 0 = \frac{12}{5}$$

c. Box II, since for Box I the highest possible net gain is \$1 with a probability of 1/3, but for Box II the highest possible gain is \$3 with probability of 1/5. Note that $V(G_I) < V(G_{II})$; i.e., Box I net gain varies less from $E(G_i) = 0$ than Box II net gain.

3.14 For estimation purposes, let x = mean age per group. Also, instead of the number of drivers per age group, let us use the percentage of drivers in each age group. Now we can estimate $\bar{x}$, s, and $\tilde{x}$.

x	Percent	Cumulative percent	x (percent)	x^2	x^2 (percent)
17	5.86	5.86	99.62	289	1,693.54
22	10.21	16.07	224.62	484	4,941.64
27	12.44	28.51	335.88	729	9,068.76
32	12.38	40.89	396.16	1,024	12,677.12
37	11.23	52.12	415.51	1,369	15,373.87
42	9.72	61.84	408.24	1,764	17,146.08
47	7.61	69.45	357.67	2,209	16,810.49
52	6.16	75.61	320.32	2,704	16,656.64
57	5.74	81.35	327.18	3,249	18,649.26
62	5.62	86.97	348.44	3,844	21,603.28
67	5.01	91.98	335.67	4,489	22,489.89
72	8.02	100.00	577.44	5,184	41,575.68
			4,146.75		198,686.25

So, $\bar{x} = \frac{4146.75}{100} = 41.47$, $s = \sqrt{\left(\frac{198686.25}{100}\right) - (41.47)^2} = \sqrt{(249.90)} = 15.81$, and $\tilde{x} = 37$

(Answers may vary).

3.15 $E(\text{number of sales}) = 0 \cdot p(0) + 1 \cdot p(1) + 2 \cdot p(2)$

$= 0(0.7) + 1(0.2) + 2(0.1) = 0.4$

$V(\text{number of sales}) = 0^2 \cdot p(0) + 1^2 \cdot p(1) + 2^2 \cdot p(2) - [E(\text{number of sales})]^2$

$= 0(0.7) + 1(0.2) + 4(0.1) - (0.4)^2 = 0.44$

Standard deviation of sales $= \sqrt{V(\text{sales})} = \sqrt{0.44} = 0.6633$