HW#4 MthSC 400

3.23 **a.** $P(X=2) = \binom{4}{2}(0.2)^2(0.8)^2 = 0.1536$

b. $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$

$$= \sum_{x=2}^{4} \binom{4}{x}(0.2)^x(0.8)^{4-x}$$

$$= \binom{4}{2}(0.2)^2(0.8)^2 + \binom{4}{3}(0.2)^3(0.8) + \binom{4}{4}(0.2)^4$$

$$= 0.1536 + 0.0256 + 0.0016 = 0.1808.$$

c. $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$$= \sum_{x=0}^{2} \binom{4}{x}(0.2)^x(0.8)^{4-x}$$

$$= \binom{4}{0}(0.8)^4 + \binom{4}{1}(0.2)^1(0.8)^3 + \binom{4}{2}(0.2)^2(0.8)^2$$

$$= 0.4096 + 0.4096 + 0.1536 = 0.9728$$

d. $E(X) = np = 4(0.2) = 0.8$

e. $V(X) = np(1-p) = 4(0.2)(0.8) = 0.64$

3.26 Let X = number of surviving fish. Then X has a binomial distribution with parameters $n = 20$, $p = 0.8$.

a. $P(X = 14) = P(X \leq 14) - P(X \leq 13) = 0.196 - 0.087 = 0.109$

b. $P(X \geq 10) = 1 - P(X \leq 9) = 1 - 0.001 = 0.999$

c. $P(X \leq 16) = 0.589$

3.33 Let X = number of radar sets out of n that detect an intruding aircraft. Then X has a binomial distribution with parameters n, and $p = 0.9$.

a. $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{2}{0}(0.9)^0(0.1)^2 = 0.99$

b. $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{4}{0}(0.9)^0(0.1)^4 = 0.9999$

3.34 $P(\text{detection}) = 1 - P(X=0) = 1 - \binom{n}{0}(0.9)^0(0.1)^n = 0.99$; i.e., $(0.1)^n = 0.01$, i.e.,

$$n = \frac{\ln(0.01)}{\ln(0.1)} = 2.$$

3.35 Let X = number of components out of the four that last longer than 1000 hours. The probability that a given component lasts longer than 1000 hours is 0.8; thus X has a binomial distribution with parameters $n = 4$, $p = 0.8$.

a. $P(X = 2) = \binom{4}{2}(0.8)^2(0.2)^2 = 0.1536$

b. $P(X \geq 2) = 1 - P(X \leq 1) = 1 - \binom{4}{0}(0.8)^0(0.2)^4 - \binom{4}{1}(0.8)^1(0.2)^3$

$= 1 - 0.0016 - 0.0256 = 0.9728$

3.37 Y has a binomial distribution with parameters $n = 4$, $p = 0.1$.

$$E(C) = E(3Y^2 + Y + 2) = 3E(Y^2) + E(Y) + 2$$

$$= 3(V(Y) + [E(Y)]^2) + E(Y) + 2 = 3np(1-p) + 3(np)^2 + np + 2$$

$$= 3(4)(0.1)(0.9) + 3[(4)(0.1)]^2 + 4(0.1) + 2 = 3.96$$