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# HW #5 MthSc 400

**3.40** **a.** $P(Y \geq 2) = 1 - P(Y = 1) = 1 - p(1-p)^0 = 1 - p = 0.9$

**b.** $$P(Y > 4 | X > 2) = \frac{P(Y>4, Y>2)}{P(Y>2)} = \frac{P(Y>4)}{P(Y>2)} = \frac{1-P(Y \leq 4)}{1-P(Y \leq 2)}$$

$$= \frac{1 - p(1-p)^0 - p(1-p)^1 - p(1-p)^2 - p(1-p)^3}{1 - p(1-p)^0 - p(1-p)^1}$$

$$= 1 - \frac{p(1-p)^2 + p(1-p)^3}{(1-p)^2} = (1-p)^2$$

Note that $(1-p)^2 = 1 - p - p(1-p) = 1 - P(Y=1) - P(Y=2) = 1 - P(Y \leq 2) = P(Y > 2)$; i.e., $P(Y > 4 | Y > 2) = P(Y > 2)$.

**3.41** **a.** $P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - P(Y = 2) - P(Y = 3)$

$$= 1 - \binom{2-1}{2-1}(0.4)^2(0.6)^0 - \binom{3-1}{2-1}(0.4)^2(0.6)^1$$

$$= 1 - 0.16 - 2(0.4)^2(0.6) = 0.648$$

**b.** $P(Y = y)$ is non-zero only for $y = r, r+1, r+2, \ldots$ Therefore, for $r = 4$,

$$P(Y \geq 4) = \sum_{y=4}^{\infty} p(y) = 1.$$

**3.42** Let $Y$ = the number of the trial on which the first nondefective engine is found. Then $Y$ has a geometric distribution with parameter $p = 0.9$, and $P(Y = 2) = p(2) = (1-p)^1 p = (0.1)(0.9) = 0.09$.

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**3.45** **a.** Let $Y$ be defined as in Exercise 3.42. Then $Y$ has a geometric distribution with $p = 0.9$, and

$$E(Y) = \frac{1}{p} = \frac{10}{9}$$

$$V(Y) = \frac{1-p}{p^2} = \frac{0.1}{(0.9)^2} = 0.1235.$$

**b.** Let $X$ be defined as in Exercise 3.43. Then $X$ has a negative binomial distribution with parameters $p = 0.9$, $r = 3$, and

$$E(X) = \frac{r}{p} = \frac{30}{9} = 3.33$$

$$V(X) = \frac{r(1-p)}{p^2} = \frac{0.3}{(0.9)^2} = 0.3704.$$

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**3.46** Let $X$ = number of employees to be tested in order to find three positives. Then $X$ has a negative binomial distribution with parameters $p = 0.4$, $r = 3$, and

$$P(X = 10) = \binom{10-1}{3-1}(0.4)^3(1-0.4)^7 = 0.0645.$$

**3.47** Let total cost = $C$. Then $C = 20X$, and

$$E(C) = 20E(X) = 20\left(\frac{r}{p}\right) = 20\left(\frac{3}{0.4}\right) = 150$$

$$V(C) = V(20X) = 20^2 V(X) = 400\frac{r(1-p)}{p^2} = 400\frac{(3)(0.6)}{(0.4)^2} = 4,500$$

Standard deviation of $C = \sqrt{V(C)} = \sqrt{4,500} = 67.082$