HW #6 (MthSc 400).

3.54 a. $P(Y=4)=p(4) = \frac{\lambda^4}{4!}e^{-\lambda} = \frac{2^4}{4!}e^{-2} = 0.0902$

b. $P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - F(3) = 1 - 0.857 = 0.143$

c. $P(Y < 4) = P(Y \leq 3) = F(3) = 0.857$

d. $P(Y \geq 4 | Y \geq 2) = \frac{P(Y \geq 4, Y \geq 2)}{P(Y \geq 2)} = \frac{P(Y \geq 4)}{P(Y \geq 2)} = \frac{1 - P(Y \leq 3)}{1 - P(Y \leq 1)} = \frac{1 - F(3)}{1 - F(1)}$

$= \frac{1 - 0.857}{1 - 0.406} = 0.2407$

3.55 Let Y = number of calls arriving in a given one-minute period. Then Y has a Poisson distribution with parameter $\lambda = 4$.

a. $P(Y=0) = p(0) = \frac{4^0}{0!}e^{-4} = e^{-4} = 0.0183$

b. $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - F(1) = 1 - 0.092 = 0.908$

c. Let X = number of calls arriving in a given two-minute period. Then X has a Poisson distribution with parameter $\lambda = 2(4) = 8$, and $P(X \geq 2) = 1 - F(1) = 1 - 0.003 = 0.997$.

3.56 Let Y = number of missing pulses per disk. Then Y has a Poisson distribution with parameter $\lambda = 0.1$.

a. $P(Y=0) = F(0) = 0.905$

b. $P(Y > 1) = 1 - P(Y \leq 1) = 1 - F(1) = 1 - 0.995 = 0.005$

c. Let X = number of missing pulses on two disks. Then X has a Poisson distribution with parameter $\lambda = 2(0.1) = 0.2$, and $P(X = 0) = F(0) = 0.819$. Also, note that P(2 out of 2 disks have no missing pulses) $= \binom{2}{2}$ [P(a given disk has no missing pulses)]$^2 = (0.905)^2 = 0.819$.

3.61 Let Y = number of customer arrivals in a given hour. Then Y has a Poisson distribution with $\lambda = 8$.

a. $P(Y=8) = P(Y \leq 8) - P(Y \leq 7) = 0.593 - 0.453 = 0.140$

b. $P(Y \leq 3) = 0.042$

c. $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - 0.003 = 0.997$

3.63 **a.** Let X = number of customers that arrive in a given two-hour period of time. Then X has a Poisson distribution with $\lambda = 2(8) = 16$ and

$$P(X=2) = \frac{16^2}{2!}e^{-16} = 128e^{-16} = 1.44 \times 10^{-5}.$$

b. The two one-hour time periods are nonoverlapping, and therefore X = total number of customers that arrive in the given two-hour time period has a Poisson distribution with $\lambda = 2(8) = 16$, and, as for part (a), $P(X=2) = 144 \times 10^{-5}$.

Consistent with this answer, note the following. Let Y_1 = number of customers that arrive between 1:00 p.m. and 2:00 p.m. and Y_2 = number of customers that arrive between 3:00 p.m. and 4:00 p.m. Then Y_1 and Y_2 are each distributed as Poisson with $\lambda = 8$ and

$$\begin{aligned}
P(Y_1 + Y_2 = 2) &= P(Y_1 = 0, Y_2 = 2) + P(Y_1 = 1, Y_2 = 1) \\
&\quad + P(Y_1 = 2, Y_2 = 0) \\
&= 2 \cdot p(0)p(2) + [p(1)]^2 \\
&= 2\left(\frac{8^0}{0!}e^{-8}\right)\left(\frac{8^2}{2!}e^{-8}\right) + \left(\frac{8^1}{1!}e^{-8}\right)^2 \\
&= 7.2 \times 10^{-6} + 7.2 \times 10^{-6} = 1.44 \times 10^{-5}.
\end{aligned}$$

3.68 Let Y = number of breakdowns in time = t hours. Then Y has a Poisson distribution with $\lambda = 0.2t$ and

$$\begin{aligned}
E(R) &= E(200t - 50Y^2) = 200t - 50E(Y^2) = 200t - 50(V(Y) + [E(Y)]^2) \\
&= 200t - 50[0.2t + (0.2t)^2] = 190t - 2t^2.
\end{aligned}$$

To obtain the maximum R, solve $\frac{dE(R)}{dt} \overset{\text{set}}{=} 0$; i.e.,

$$\frac{d(190t_0 - 2t_0^2)}{dt} = 190 - 4t_0 = 0 \Rightarrow t_0 = 47.5.$$

Note that this is a maximum since $\frac{d^2E(R)}{dt^2} = -4 < 0$, so the required maintenance interval, t_0, is 47.5 hours.