Azimuth, Elevation, and Coupling Compensation for the Tendril

by Lara Cowan

I. Introduction

The Tendril is a device consisting of tendons, springs, motors, and a pulley system. The pulley system controls lines which act as tendons for the Tendril. As the lines are pulled, the Tendril moves in a given direction. The top joint is offset from the bottom joint by 45° (CCW). The motor encoder values are represented by m_0, m_1, m_2, and m_3 for the four motors. $Joint_1$ (top) consists of m_0 and m_1 while $joint_0$ (tip) uses m_2 and m_3. The encoder values are the values input into the interface to move the Tendril. The motor encoders check the values and stop when the measured value is within error tolerance of the input value. When calculating the needed encoder values some assumptions must be made. It is assumed that the motors are balanced. This means that the origin is at (0,0) for both joints. The origin is the center of the encoder values for each joint. This means that if a motor is set to x, then -x will give the same elevation at an azimuth offset by 180°. The maximum motor values are equal for both joints ($m_{0max} = m_{1max}$, $m_{2max} = m_{3max}$). These are the values each motor needs to be given to individually raise to a given elevation.

II. Azimuth

Azimuth is the angle made between the joint and a plane parallel to the wall. The azimuth of the joints is measured CCW with 0° being parallel to the wall as in Figure 1. The motors are attached with an offset of 90° at each joint. The motors are set at differing azimuths: $motor_0$ is at 135°, $motor_1$ is at 45°, $motor_2$ is at 0°, and $motor_3$ is at 90°. Figure 1 is a unit circle showing the azimuth as seen from above. The azimuth of the joints and the corresponding encoder values are shown in Figures 2 and 3 for increments of 45°.

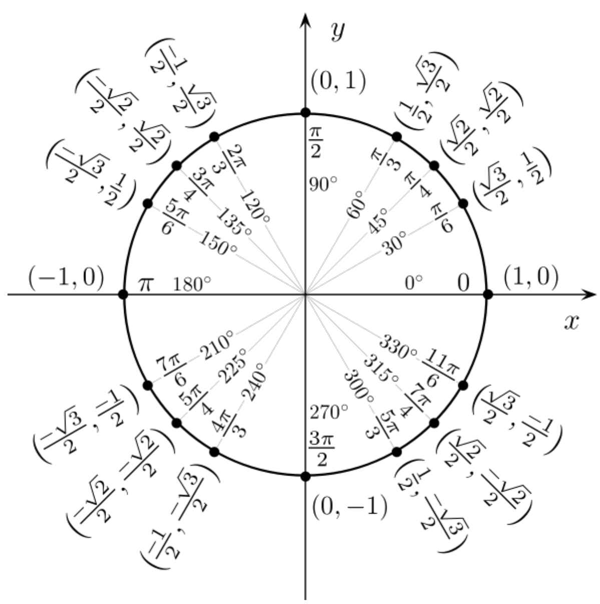

Figure 1: Unit Circle Showing Azimuth (as seen from above)

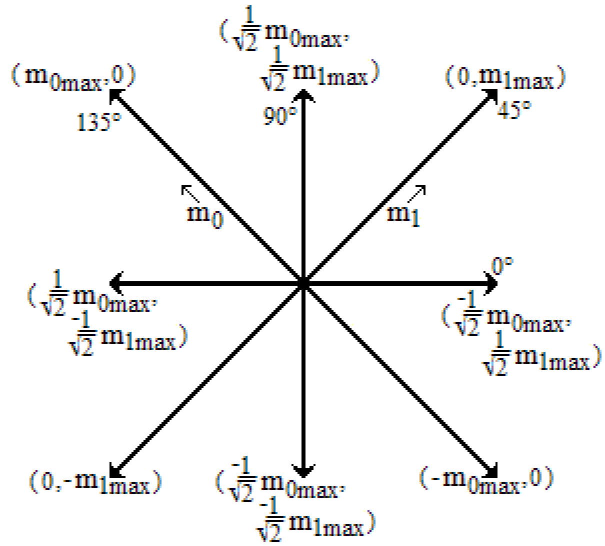

Figure 2: Joint$_1$ Encoder Values at 45° Increments

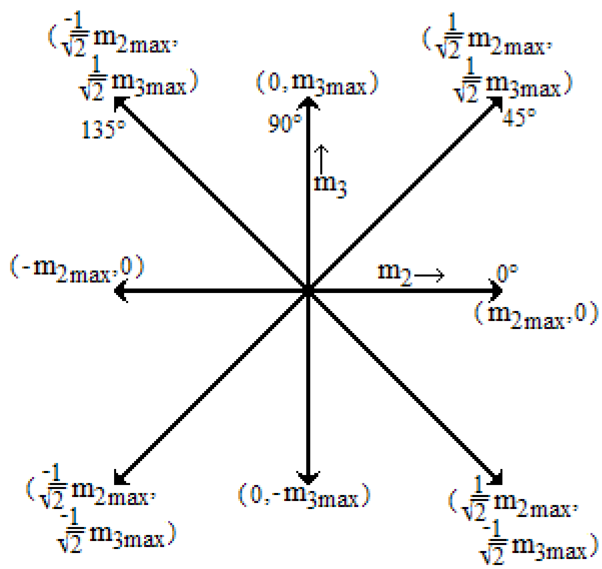

Figure 3: Joint$_0$ Encoder Values at 45° Increments

The elevation determines the maxim encoder values of the motors. Figure 4 shows the elevation vs encoder values that were graphed in Matlab. The data indicates that the function $m_{max}(el)$ follows a linear function up to a certain point. The tip joint elevation is a more linear function than the top joint.

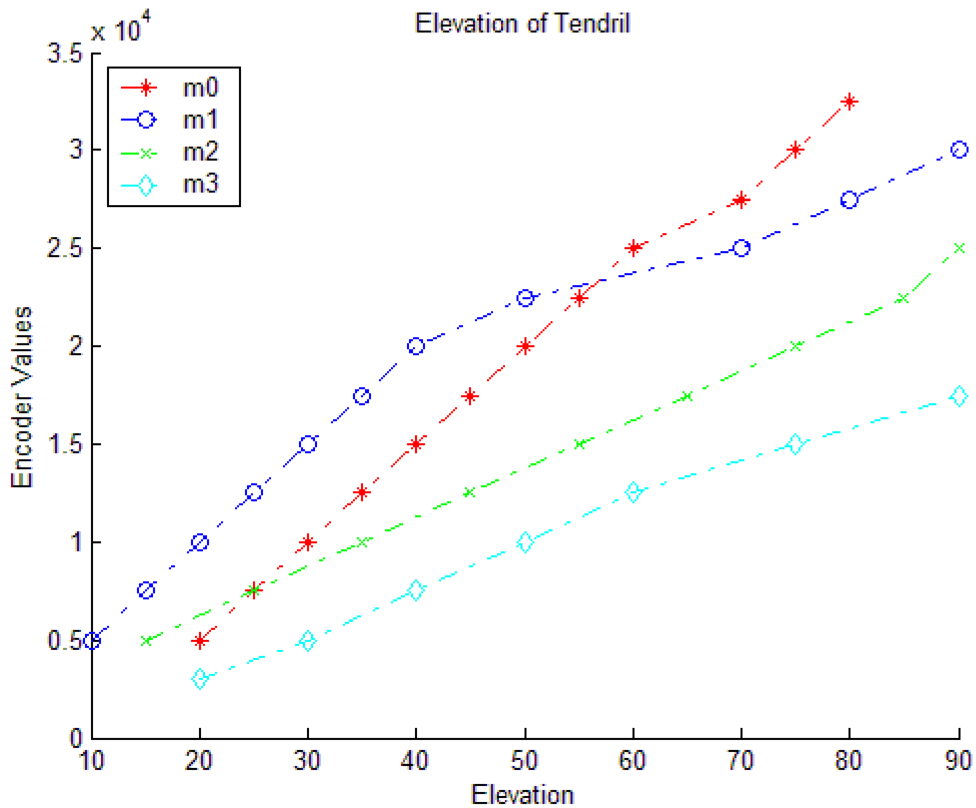

Figure 4: Encoder Values vs Elevation

The azimuth is used, along with the maximums, to find the encoder values to send to the motors. The plot of azimuth vs encoder values reveals that the function is sinusoidal. First we shall assume that only one joint is moved at a time. If only $joint_1$ is moved, then equation 1 can be used to calculate the encoder values.

$$\begin{aligned} m_{0\max}(el_1) &= m_{1\max}(el_1) = m_{j1} \\ m_0 &= m_{j1} \cdot \sin(az_1 - 45°) \\ m_1 &= m_{j1} \cdot \cos(az_1 - 45°) \end{aligned} \quad (1)$$

If $joint_0$ is moved, then $joint_1$ must compensate to stay in position using formulas 2 and 3. The coupling compensation function $f(el_0)$ is derived in section IV and depends on the elevation of $joint_0$.

$$\begin{aligned} m_{0\max}(el_1) &= m_{1\max}(el_1) = m_{j1} \\ m_0 &= m_{j1} \cdot \sin(az_1 - 45°) - f(el_0) \cdot \sin(az_0 - 45°) \\ m_1 &= m_{j1} \cdot \cos(az_1 - 45°) - f(el_0) \cdot \cos(az_0 - 45°) \end{aligned} \quad (2)$$

$$\begin{aligned} m_{2\max}(el_0) &= m_{3\max}(el_0) = m_{j0} \\ m_2 &= m_{j0} \cdot \cos(az_0) \\ m_3 &= m_{j0} \cdot \sin(az_0) \end{aligned} \quad (3)$$

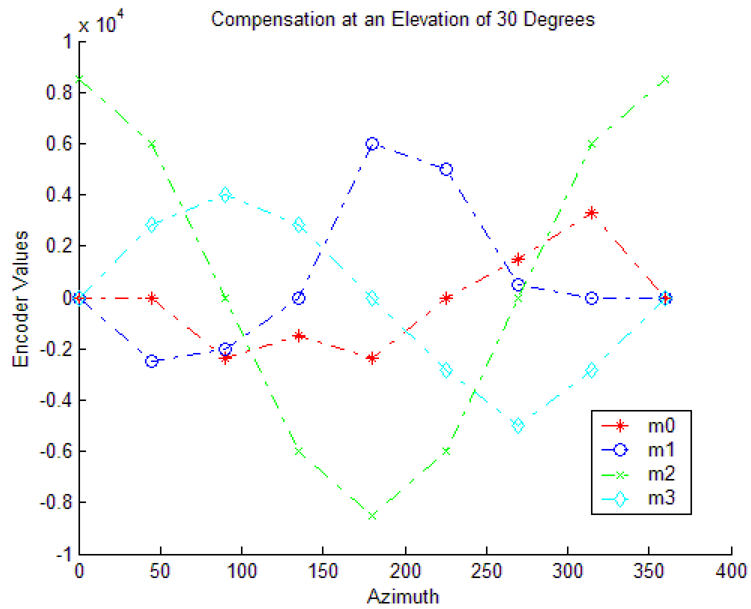

Figure 5: Azimuth vs Encoder Values at 30° Elevation

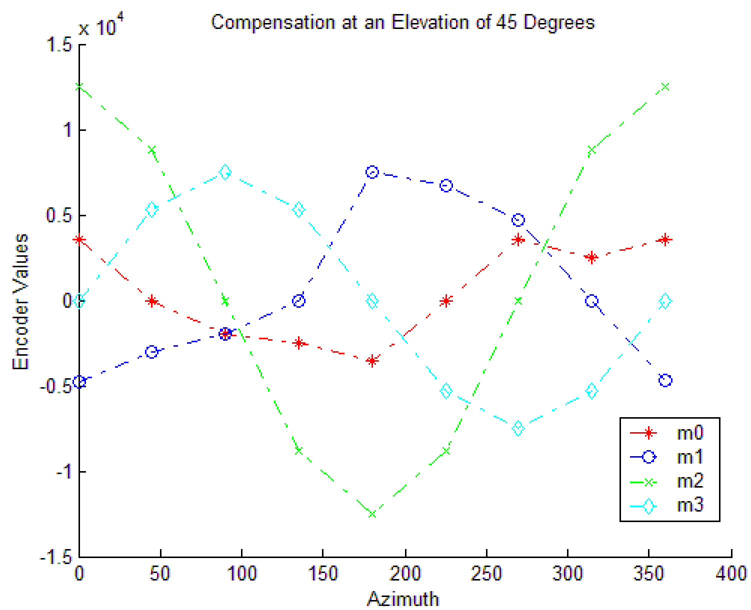

Figure 6: Azimuth vs Encoder Values at 45° Elevation

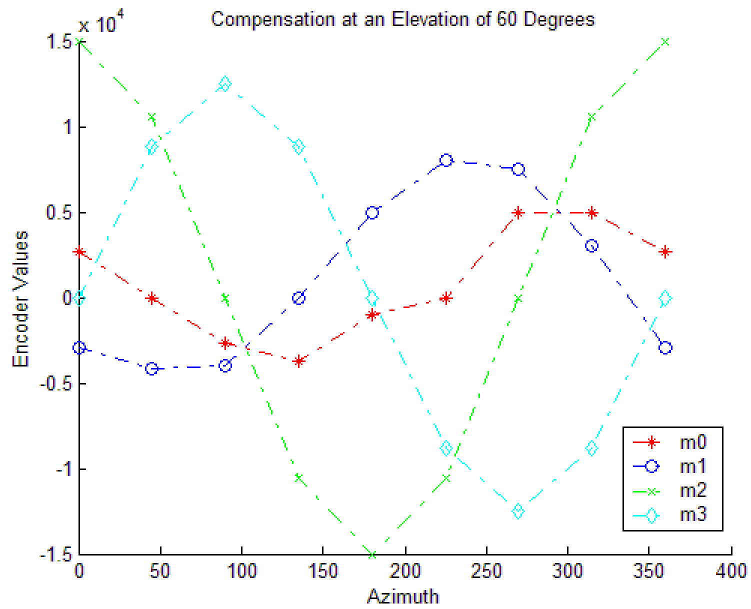

Figure 7: Azimuth vs Encoder Values at 60° Elevation

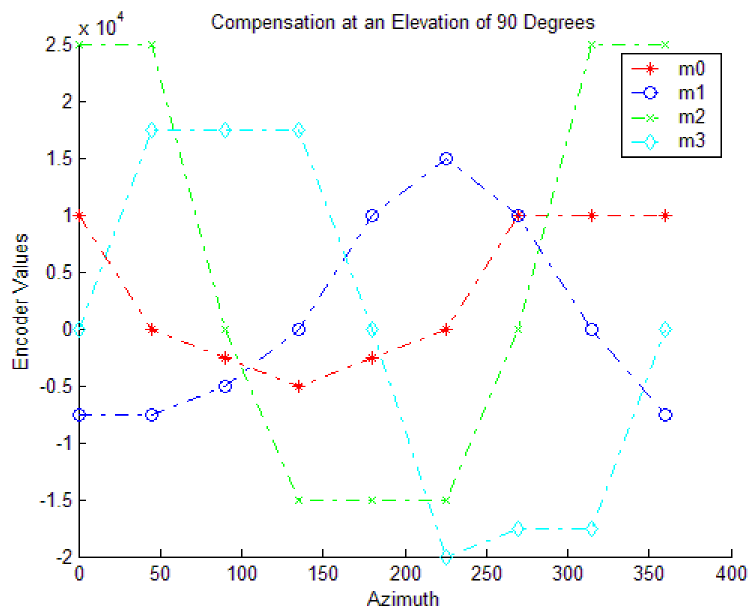

Figure 8: Azimuth vs Encoder Values at 90° Elevation

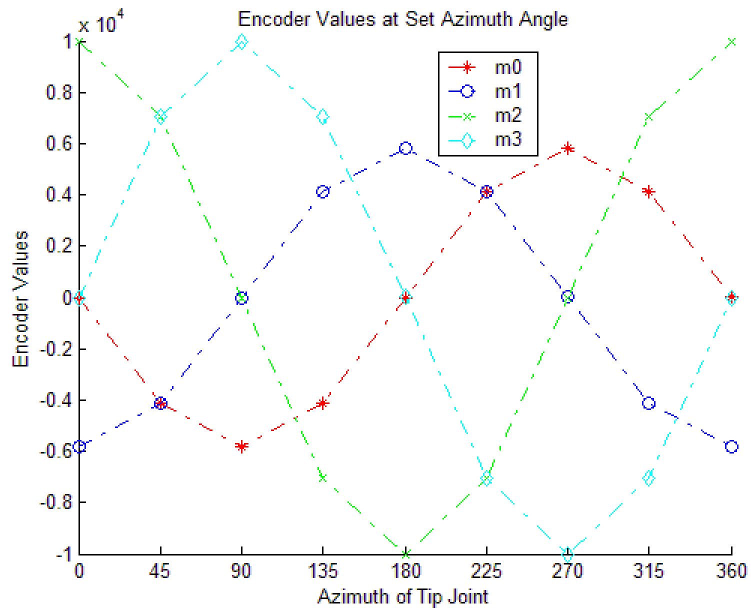

Figure 9: Azimuth vs Encoder Values at 45° Elevation using Formula

Figures 5 through 8 are graphs of the azimuth vs encoder values at varying elevations. The tip joint's motors follow a sinusoid quite closely. The top joint's motors approximate a sinusoid with lessening error as the elevation increases. Figure 9 shows the encoder values derived from equations 2 and 3 using balanced motors with $m_{j1} = 17500$ and $m_{j0} = 10000$, which are the averages of the values measured at an elevation of 45°. The formula does not match the data exactly for many reasons. The Tendril is not an ideal machine. The encoders are not centered nor balanced. A more precise function to calculate encoder values can be found by using an equation for an off-center ellipse instead of a centered circle to model the azimuth movement. A mathematically perfect Tendril would use a circle for the comparison of the motors, but the actual Tendril's plot tends to square out as the elevation increases as can be seen from Figures 10 and 11. The encoder for motor 1 is off-center by about 500. As the elevation increases, the top joint stays centered around the same rough spot. The center for the bottom joint seems to move off to the right at increasing elevations after 60°.

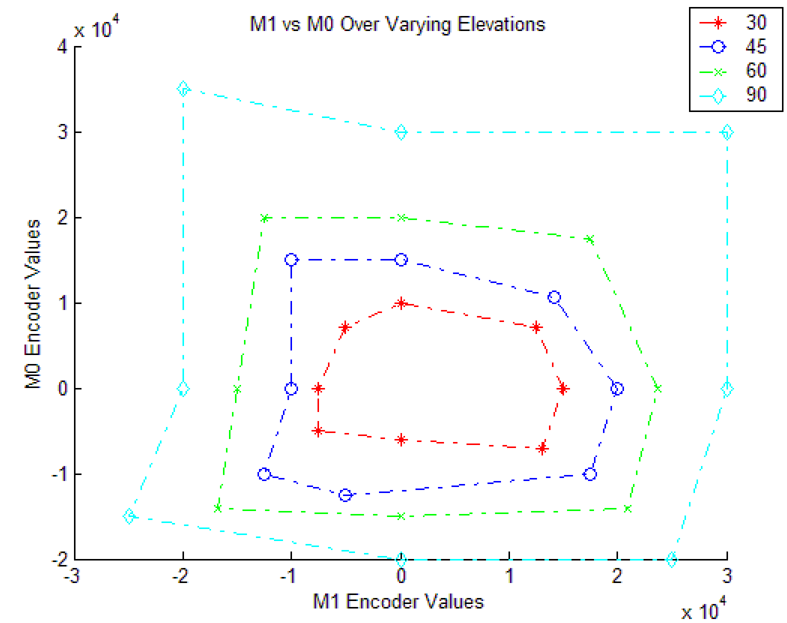

Figure 10: m1 vs m0

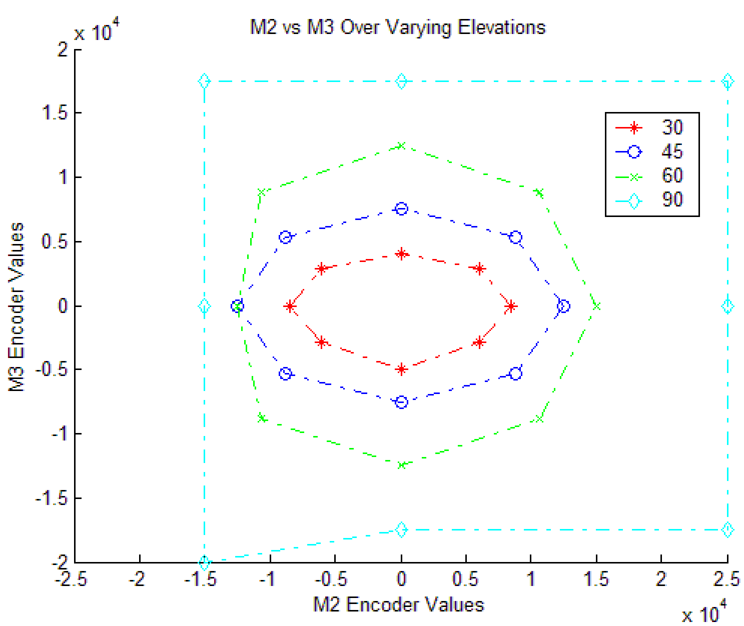

Figure 11: m2 vs m3

These graphs show that the tip joint is better behaved than the top joint. This makes sense since the tip joint does not have the extra string running through it as the top joint does. Using more precise equations can fix some of the problems. The maximum encoder values follow a slight ellipse that squares out as the elevation increases. This is likely due to the mechanical measures used to tighten the Tendril to remove the slack from the lines. When the Tendril is in the initial position, with both joint straight down, the lines must be drawn in for a small time to remove the slack before it will move. Similarly, as the elevation approaches 90°, it is harder for the lines to be drawn in by the pulleys. This means that the Tendril slows slightly and needs a higher encoder value to reach its desired position. A nonlinear formula using both azimuth and elevation would be needed to fix this.

III. Elevation

The next section deals with the elevation formula. The maximum motor encoder values, $m_{max}(el)$, used in equations 1, 2, and 3 are derived from the desired elevation of the joints. First there are some assumptions to be made. The first assumption is that there is no gravity. Therefore the Tendril will not bend or distort under its own weight. The second assumption is that when the Tendril bends it will make an undistorted arc. In real world circumstances the Tendril will bend differently because of gravity. The third assumption is that the springs are already compressed. If the springs are not compressed then it will take a finite amount of time for the spring to compress before it bends away from the straight position. The initial position of the Tendril is hanging straight down with the elevation set to 0°. The maximum advisable elevation is 90° but the Tendril can bend past that, maybe up to 135°. Figure 12 shows the elevation of the Tendril for a variety of angles. Figure 13 is a diagram of the Tendril showing the new variables for elevation.

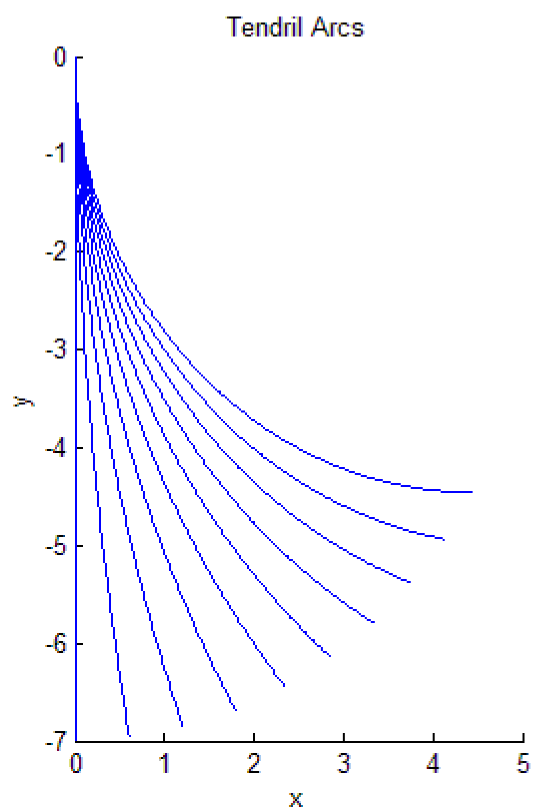

Figure 12: Elevation of a Tendril joint from 0° to 90°

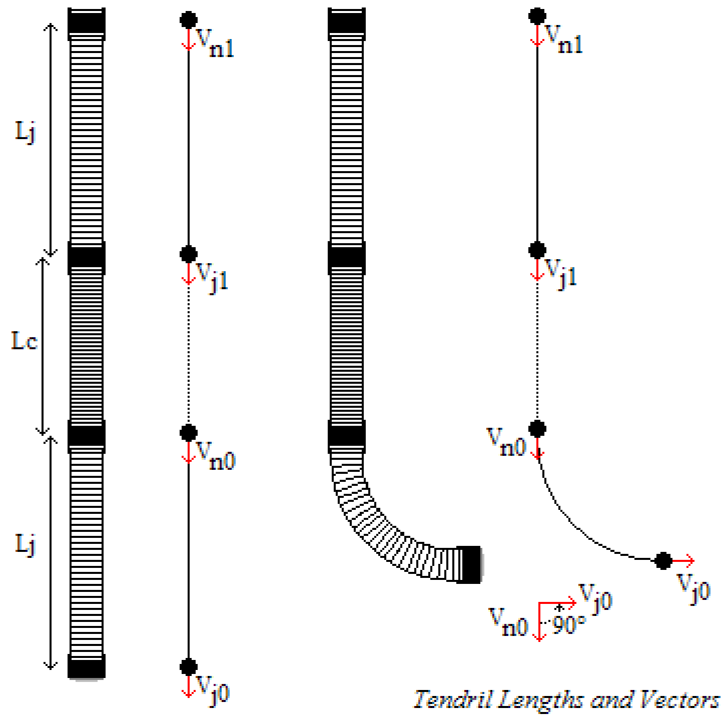

Figure 13: Diagram of Tendril and Vectors

In Figure 13, the new variables are introduced to describe the elevation of the Tendril. The lengths of the Tendril joints and connecting sections are L_j and L_c respectively. These are assumed to be equal for each joint and section. The vectors V_{n1} and V_{n0} are used to describe the direction of the top node of the joint. The vectors V_{j1} and V_{j0} are used to describe the direction of the bottom node of the joint. This is used to determine the angle that the joint is bent. The angles of elevation of the joints are el_1 and el_0. These are the angles between the top and bottom node vectors for each joint as seen in Figure 13.

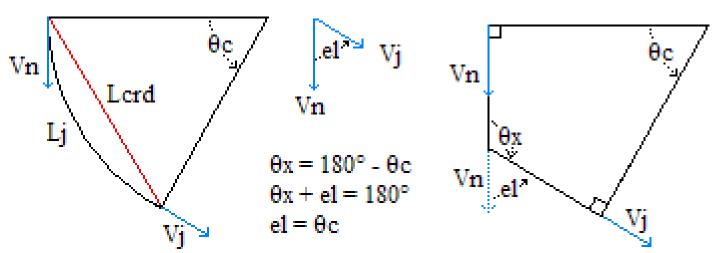

Figure 14: Angle Diagrams

As seen in Figure 14, the angle of elevation (el) is equal to the arc angle (θ_c) formed by the Tendril joint n. The chord from V_n to V_j is approximately equal to the length (L_{crd}) of the taut tendon being pulled. The difference between the actual length of the Tendril (L_j) and the taut chord is the length (ΔL) that the tendon that has been pulled. This value can be used to find m_{max}, the maximum encoder value for the specific joint elevation.

$$L_{crd} = r \cdot crd(\theta_c) = 2r \cdot \sin\left(\frac{el}{2}\right) \qquad (4)$$

$$r = \frac{180 \cdot L_j}{\pi \theta_c} = \frac{180 \cdot L_j}{\pi el} \qquad (5)$$

$$\Delta L = L_j - L_{crd} = L_j \left[1 - \frac{360}{\pi el} \cdot \sin\left(\frac{el}{2}\right) \right] \qquad (6)$$

$$m_{max} = cm2enc \cdot L_j \left[1 - \frac{360}{\pi el} \cdot \sin\left(\frac{el}{2}\right) \right] \qquad (7)$$

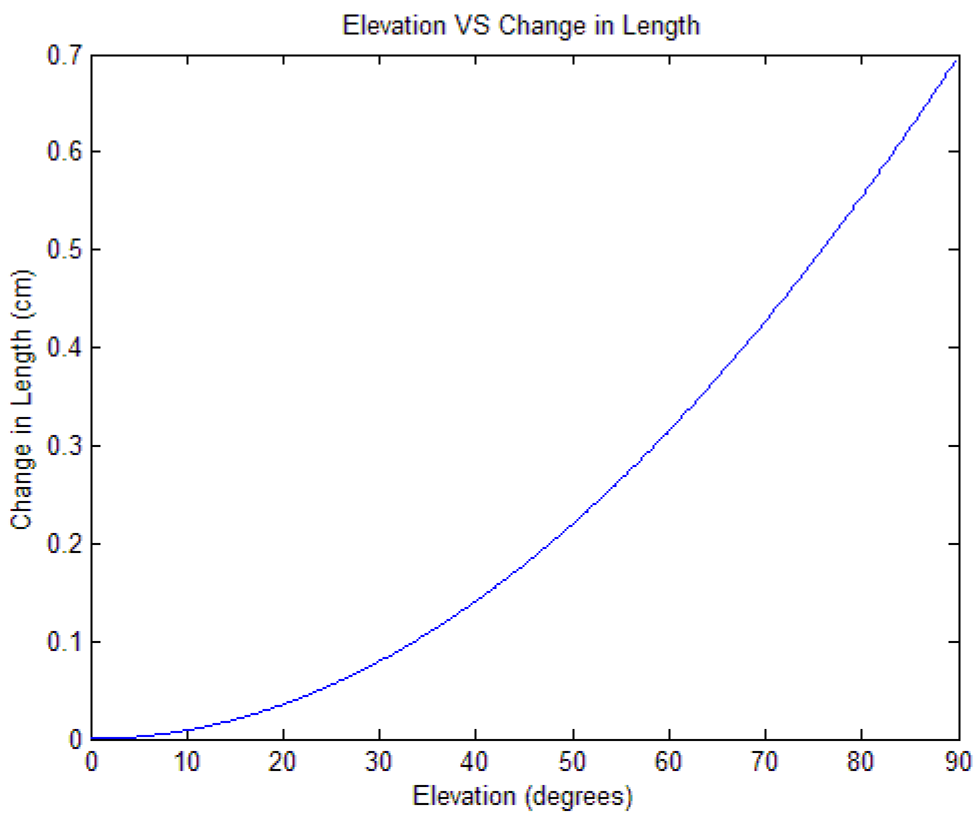

Figure 15: el vs ΔL

The formula for ΔL can be used to find the maximum encoder value required for a certain local elevation. The Tendril has an L_j of 7cm and an L_c of 5cm. The conversion from cm to encoder steps is used to change equation 6 into a formula that gives out encoder steps as seen in equation 7. The conversion number (cm2enc) was found to be 50000 encoder steps/cm for $Joint_1$ and 25000 encoder steps/cm for $Joint_0$. The disparity in value can be attributed to the slack in the line for $Joint_1$. These values will change when the Tendril setup is altered or the rate is changed. Figure 16 shows a plot of elevation and encoder values measured from the Tendril.

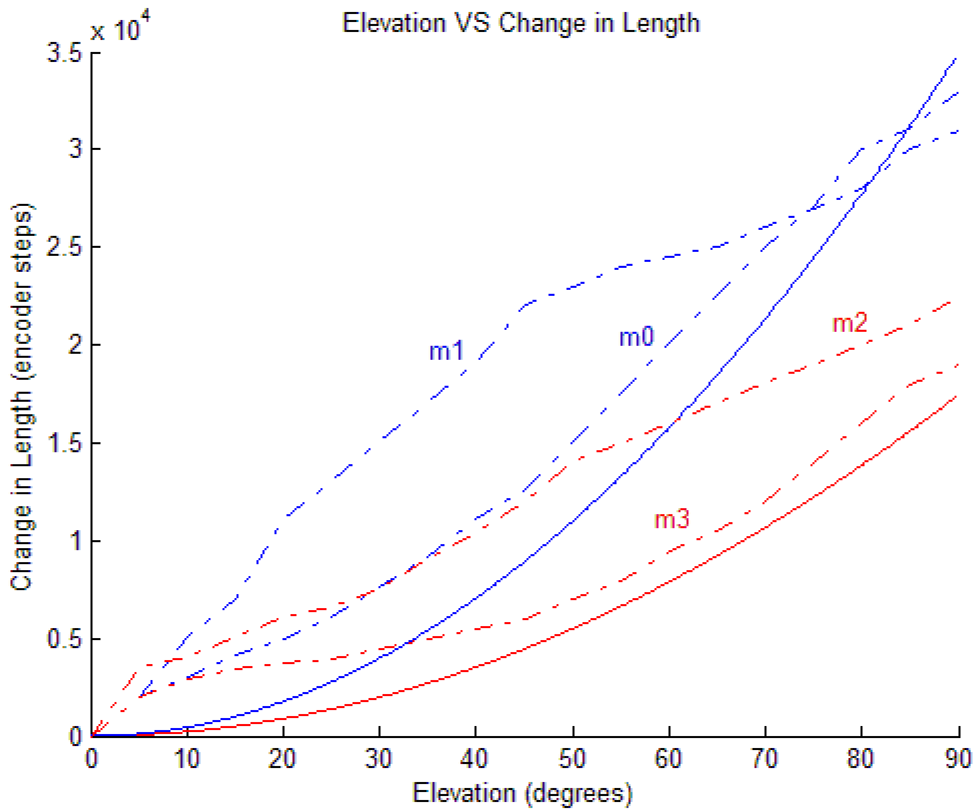

Figure 16: Encoder values at Elevation Steps of 5°

The encoder values are not the same as those predicted. Since the Tendril's motors are not balanced, the tightness of the tendons in not equal and it takes some time for the joint to move at first. When looking at the data for the first 5°, it is evident that the Tendril must take some initial steps to draw in the slack before it can start moving. Equation 7 is not precise enough so another formula to compute the maximum encoder values needs to be derived. A better way to compute the difference in the length of the tendon is to calculate the difference between the inner length and the center of the Tendril. Equation 8 shows the new equation for finding m_{max}. This is more accurate than the stylized representation in Figure 14 since it more accurately describes the change in length. Figure 17 defines the variables used in the equations, with *el* in degrees.

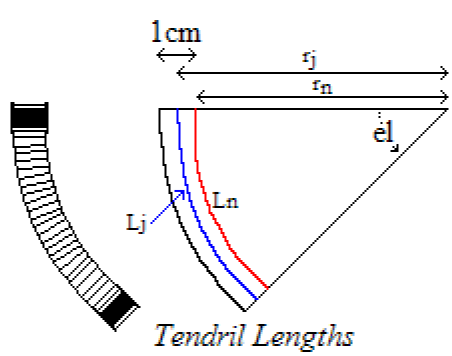

Figure 17: Diagram of Lengths

$$r_j = \frac{180 \cdot L_j}{\pi \cdot el}$$

$$r_n = r_j - 0.5$$

$$L_n = \frac{\pi\, el \cdot r_n}{180} = \frac{\pi\, el \cdot (r_j - 0.5)}{180} = \frac{\pi\, el}{180}\left(\frac{180 \mathrm{L}_j}{\pi\, el} - 0.5\right) = L_j - \frac{\pi\, el}{360} \qquad (8)$$

$$m_{max} = \Delta L = L_j - L_n = \frac{\pi\, el}{360}(cm) = \frac{\pi\, el}{180}(0.5 \cdot cm2enc)(enc\, steps)$$

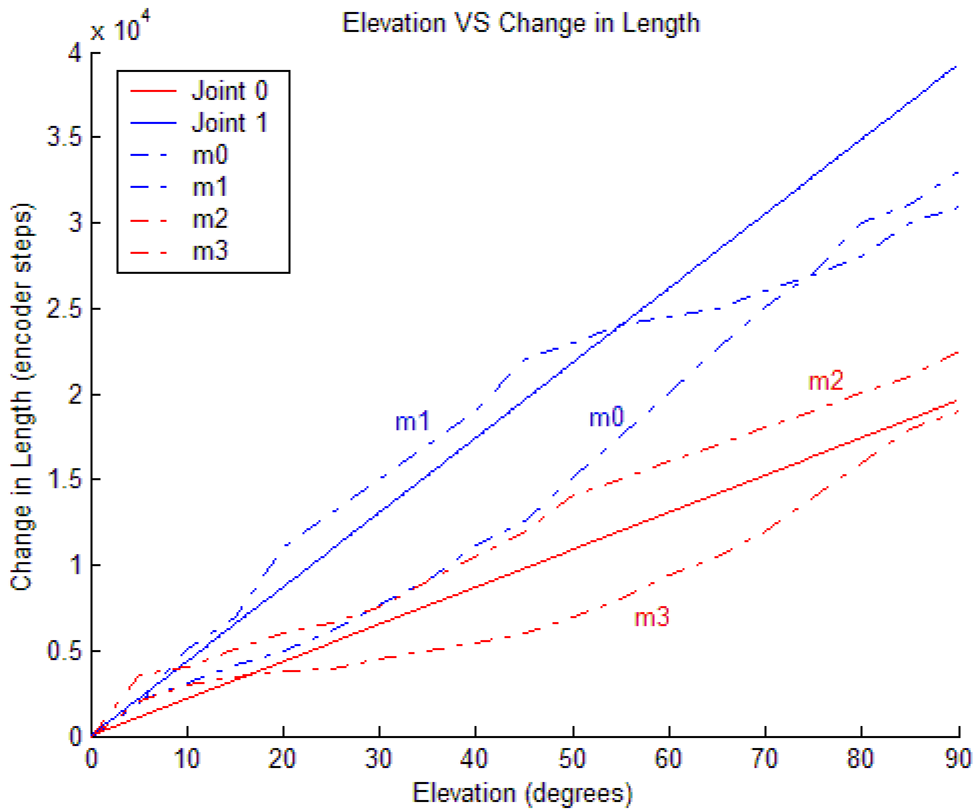

Figure 18: Graph of Elevation VS Encoder Values Using Equation 8

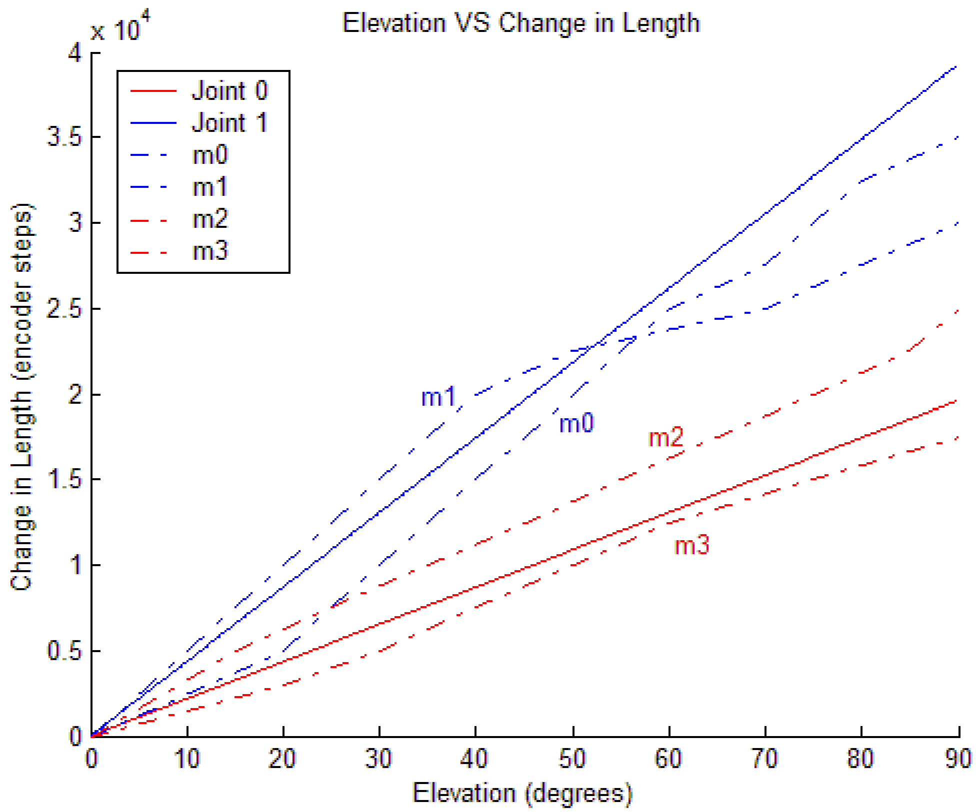

Figure 19: Using Data From Figure 4

The results from equation 8 are more balanced than equation 7. In Figure 18, the two motor encoder values more closely match the expected values for their joint. In Figure 19, the elevation data from Figure 4 is graphed against equation 8 and the results look closer. As the joints approach 90°, the springs begin to twist more as they bunch up. This causes some deviation as the elevation increases.

IV. Coupling Compensation

The elevation is the variable that is affected by joint coupling. The coupling at the joints is a large problem that needs to be resolved. When a lower joint is moved, it affects all the joints above it. Equations 9 and 10 use the elevation formula from equation 8. It is assumed that there is negligible affect from gravity. This can be simulated in experiments by placing the Tendril on its side and testing each motor individually.

$$\Delta L_0 = \left[\frac{\pi \cdot cm2enc}{360} el_0 \right] \quad (9)$$
$$\Delta L_1 = \left[\frac{\pi \cdot cm2enc}{360} el_1 \right]$$

$$m_0 = \left[\frac{\pi \cdot cm2enc}{360} el_1\right] \cdot \sin(az_1 - 45°) - \left[\frac{\pi \cdot cm2enc}{360} el_0\right] \cdot \sin(az_0 - 45°)$$
$$m_1 = \left[\frac{\pi \cdot cm2enc}{360} el_1\right] \cdot \cos(az_1 - 45°) - \left[\frac{\pi \cdot cm2enc}{360} el_0\right] \cdot \cos(az_0 - 45°)$$
$$m_2 = \left[\frac{\pi \cdot cm2enc}{360} el_0\right] \cdot \cos(az_0) \tag{10}$$
$$m_3 = \left[\frac{\pi \cdot cm2enc}{360} el_0\right] \cdot \sin(az_0)$$

The Tendril only has two joints so formulas 9 and 10 will suffice for now. If further joints are added then formulas 11 to 14 can be used to find the encoder values. It is assumed that *cm2enc* is the same for all joints. For each joint 0:n and motor x, the real encoder value (m_{Rnx}) will be calculated using the desired value (m_{Dnx}) and the previous joint's real value (m_{Rn-1x}). The function $f(el_n)$ is the elevation of joint n. The elevation equation, *f*, should work for every joint. Each of the n joints has 2 motors. The function $g_x(az_n)$ is the azimuth of motor x and depends on the orientation of the tendons for each motor. Equation 12 shows the *g* function for motors 0 to 3. Equation 13 is a simplified version of equation 10 using equations 11 and 12. The most important equation is equation 14 which describes how to get the encoder values for all motors using a summation of desired values.

$$f(el_n) = \frac{\pi \cdot cm2enc}{360} \cdot el_n \tag{11}$$

$$g_0(az_n) = \sin(az_n - 45°)$$
$$g_1(az_n) = \cos(az_n - 45°)$$
$$g_2(az_0) = \cos(az_0) \tag{12}$$
$$g_3(az_0) = \sin(az_0)$$

$$m_0 = f(el_1) \cdot g_0(az_1) - f(el_0) \cdot g_0(az_0)$$
$$m_1 = f(el_1) \cdot g_1(az_1) - f(el_0) \cdot g_1(az_0)$$
$$m_2 = f(el_0) \cdot g_2(az_0) \tag{13}$$
$$m_3 = f(el_0) \cdot g_3(az_0)$$

$$m_{Dn_x} = f(el_n) \cdot g_x(az_n)$$
$$m_{R0_x} = m_{D0_x}$$
$$m_{Rn_x} = m_{Dn_x} - m_{Rn-1_x} \tag{14}$$
$$m_{Rn_x} = \sum_{k=0}^{n} (-1)^{n+k} \cdot m_{Dk_x}$$

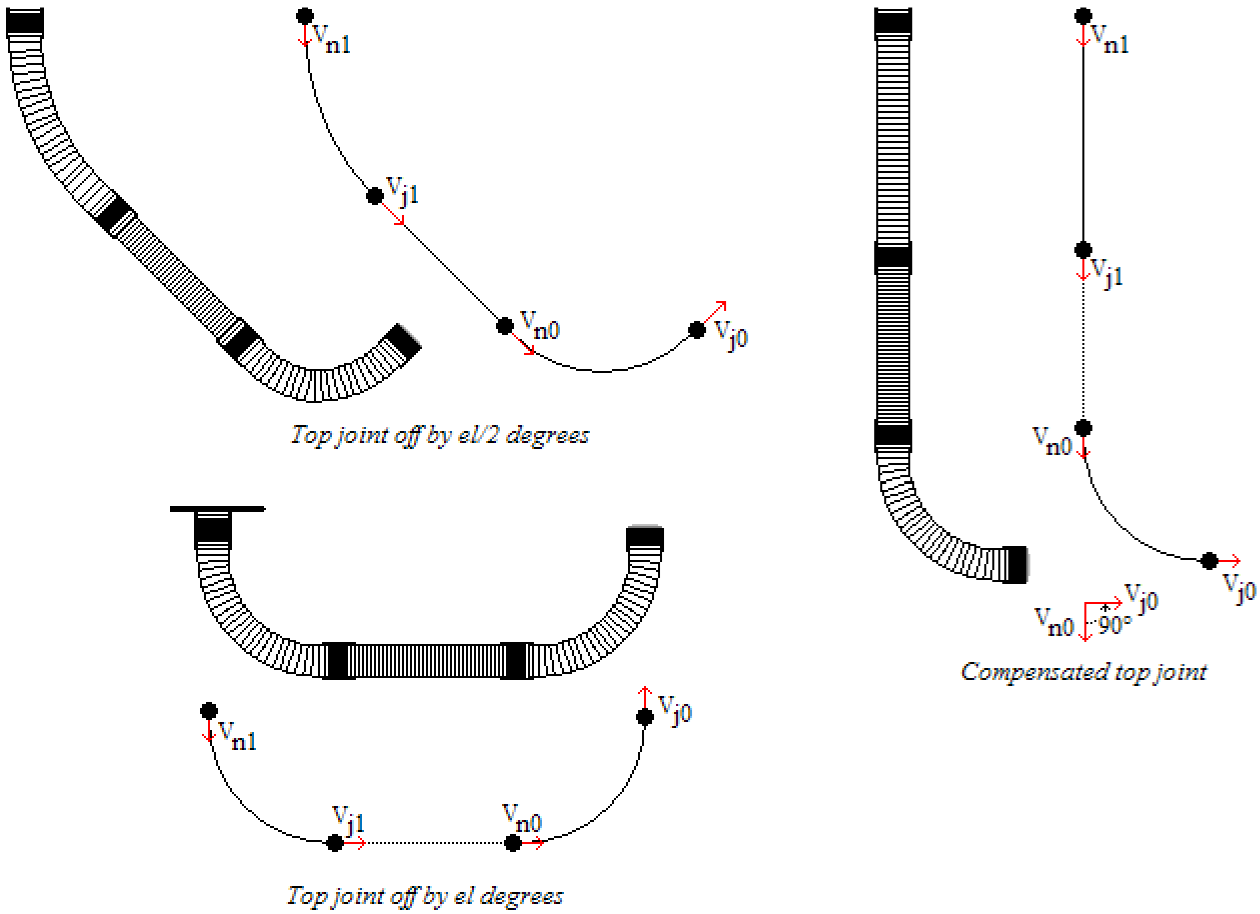

Figure 20: Compensation

The next step would be to include gravity. The connecting sections (L_c) are assumed to be too stiff to bend, but they do sag because of gravity. As you travel up the Tendril's joints, each node will be affected by gravity more than its predecessor. Since the Tendril is in effect one long spring, it can be assumed that the whole Tendril tries to bend when the lowest joint is elevated. A similar effect can be seen when placing a Slinky on a table and bending it in a circle. Since the sections bend equally, all prior joints should bend by the desired angle of the moving joint. This is not true in practice because of gravity, so new equations should be written to include gravity. Figure 21 shows a photo of the Tendril coupling problem. After developing the equations further, the next step is to move both joints and see if these coupling formulas hold.

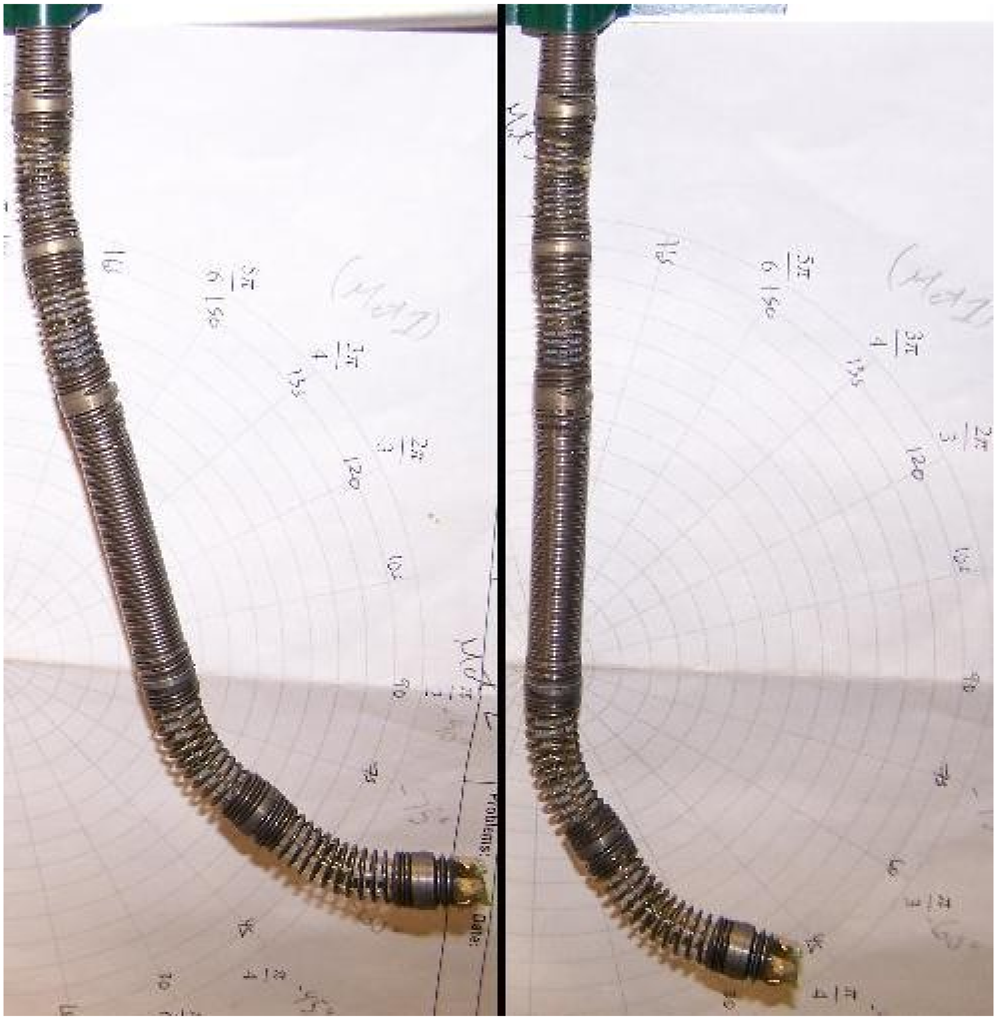

Figure 21: Uncompensated and Compensated Tendril

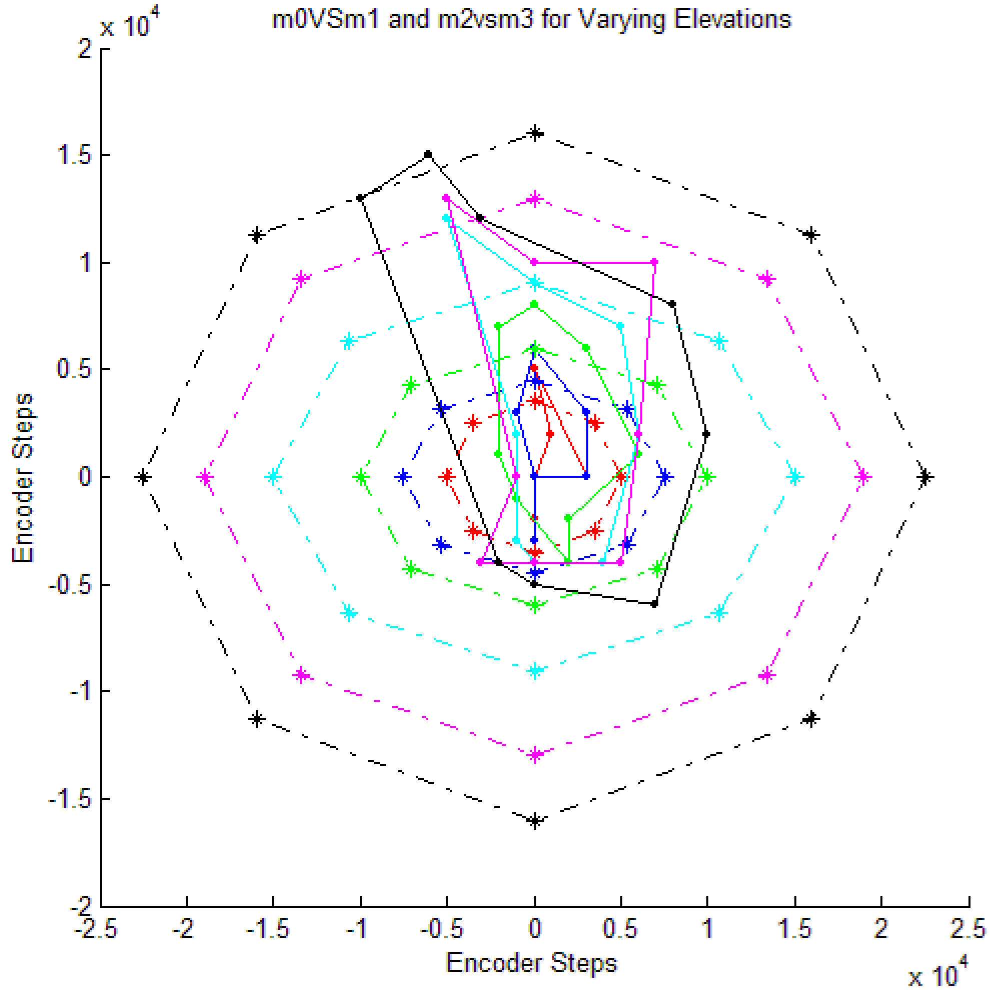

Figure 22: Joint$_1$ Compensation when Joint$_0$ is 15° to 90° in 15° Increments

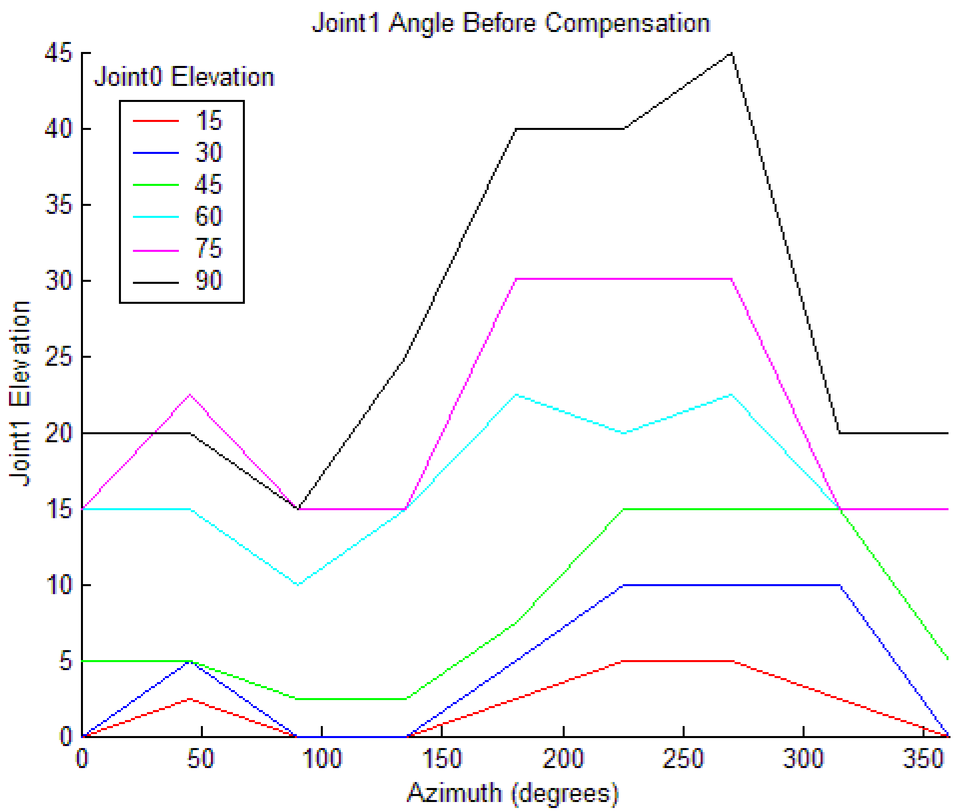

Figure 23: Elevation of Joint$_1$ Before Compensation

The compensation does not follow the circular pattern as the azimuth changes. This is because of the unbalanced and off-center motors as well as mechanical difficulties. In Figure 22, the dotted line is the value of m2 VS m3 while the solid line is m0 VS m1. The value of m1 is larger than m0, which results in an ellipse. The center is near (m0,m1) = (1000,2000) at first but it drifts towards (5000,2500) when the Joint$_0$ elevation is 90°. To fix this the Tendril would need to be restrung with all tendons having equal tautness and starting perfectly centered. Another problem, seen in Figure 23, is that the angle that Joint$_1$ is off by changes as the azimuth changes. This is because of the off-center motors. The angle should be equal across the azimuth.

The Tendril was restrung and shortened. The extra sections above the joints as well as the connecting section was removed. This leaves one connecting section and the two joints. This assembly was placed on a flat surface and the tendons were manipulated to see how it would act without gravity. Since it is a flat plane, only motor 2 was manipulated. Ignoring slight defects because of friction, the Tendril bent like the bottom image of Figure 20. The top joint bends the same angle as the bottom joint. Figure 24 shows three positions of the Tendril at around 0°, 45° and 90°. The top joint bends at the same angle. Both joints together form a partial circle.

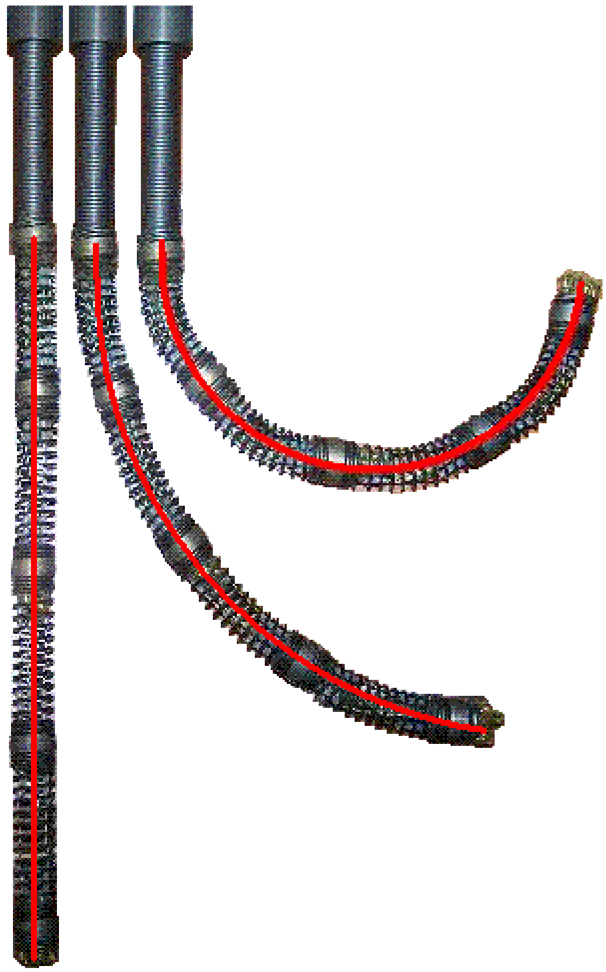

Figure 24: Tendril Bending in Semicircle

A red arc has been drawn over the Tendril in Figure 24 to show that both joints are bending equally. The tendon for motor 2 is the only one being manipulated in the two figures. When it is pulled, the whole Tendril bends in a continuous arc. Figure 25 has images of the tip joint bending from 0° to 90°. Both joints have the same elevation, which proves that the top joint bends the same elevation as the bottom joint. So Compensation should be as simple as bending the top joint in the opposite direction.

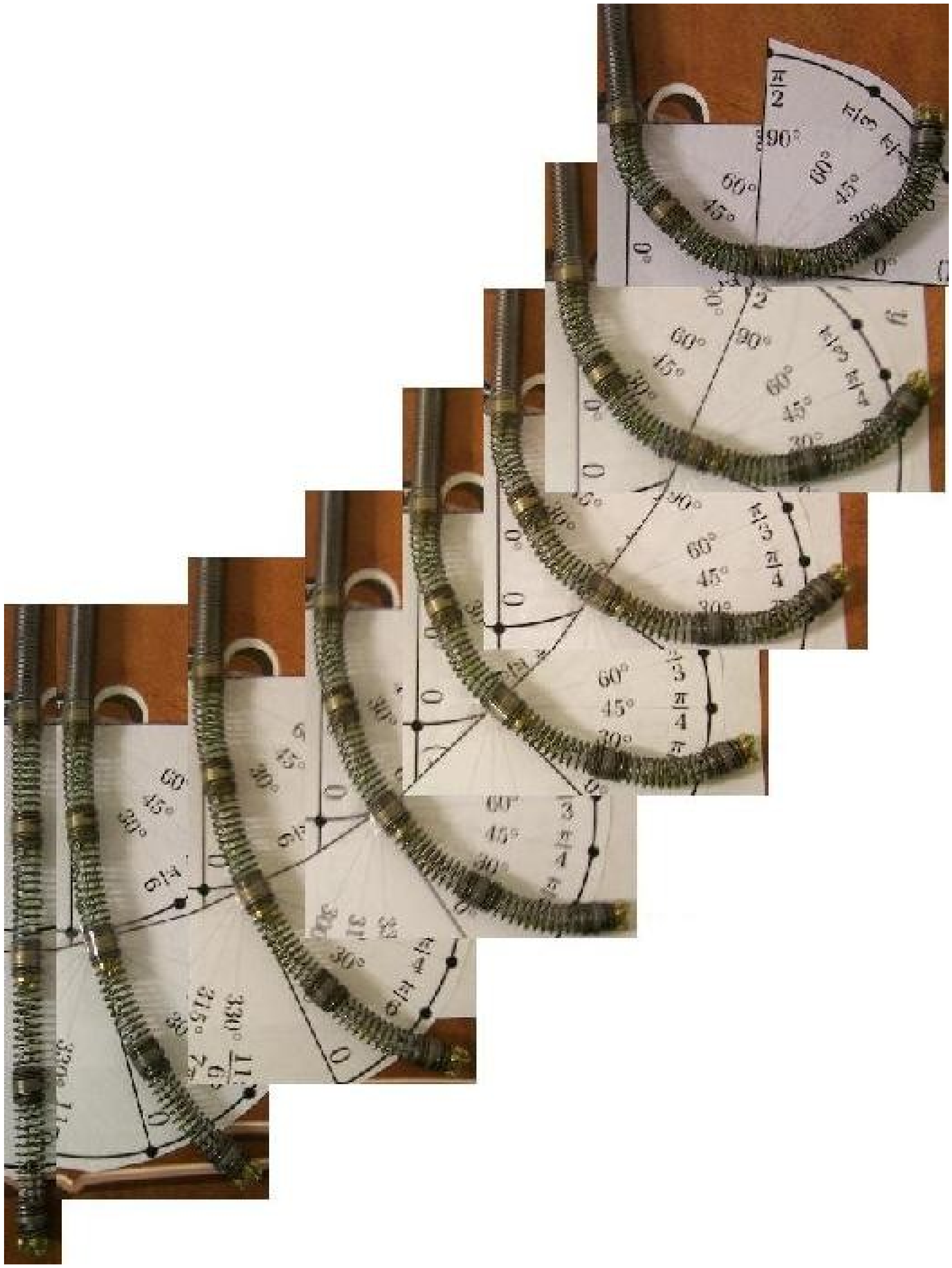

Figure 25: Various Elevations for Joint$_0$

V. Future Work

The hanging at an elevation of zero is an issue that needs to be resolved. When the Tendril passes through zero elevation, the slack must be taken up before it can move in the opposite direction. One potential solution is to increase the rate as the elevation approaches zero while decreasing the rate as the elevation increases. Another solution is to add tension springs on each of the lines to eliminate the slack. The best way to test the azimuth formulas would be at an elevation of 45° since slack and other nonlinearities can be ignored safely. The elevation formulas should be tested at the angles that the tendons attach at. Further refining of the equations should take gravity into account since it pulls on the connecting sections and affects the angle of elevation. The lower joint should be moved before the top joint so that its movement doesn't misalign the top joint. The top joint should be moved slower than the bottom joint because it has a longer arm. Software compensation for the off-center and unbalanced motors could improve performance.