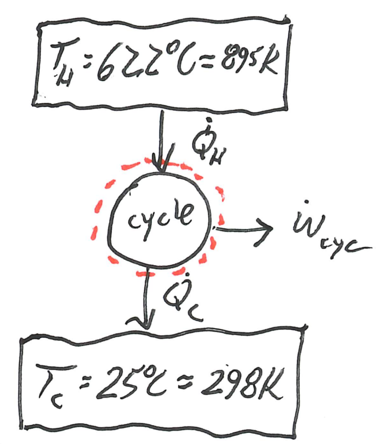
EXAM #1 – ME 3030 – Section 001 – Thermodynamics – Spring 2024
Prof. J. R. Saylor

NAME: SOLUTION

This is a closed book/closed notes exam. Use of a 4-function calculator is permitted. Zero credit will be earned for this exam if the honors pledge is not signed.

1. (10 points) Consider a coal-fired power plant that rejects heat to the environment at 25° and where the coal burns at a temperature of 622°C. The plant has an efficiency that is 61% of a perfectly reversible power plant. The plant rejects heat to the environment at a rate of 485 MW. What is the *work* in MJ delivered by the power plant during the course of a year if the plant is operational 48 weeks per year.

DIAGRAM



GIVEN: $T_H, T_C, \eta = 0.61\eta_{max}, \dot{Q}_C = 485MW$

FIND: W_{cyc} = ? MJ for 48 weeks

ASSUME:

ANALYSIS: $\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{298K}{895K} = 0.667$

$\eta = 0.61\,\eta_{max} = 0.61(0.667) = 0.407$

$\eta = \frac{\dot{W}_{cyc}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_C}{\dot{Q}_H} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H} = 1 - \frac{485MW}{\dot{Q}_H} = 0.407$

$\dot{Q}_H = 818MW$ $\quad \dot{W}_{cyc} = \dot{Q}_H - \dot{Q}_C = 818MW - 485MW$

$\dot{W}_{cyc} = 333\ MW$

For constant $\dot{W}$, $W_{cyc} = \dot{W}_{cyc}(time)$

$W_{cyc} = (333\ \frac{MJ}{s})(48wks)(7\ days/wk)(24\ hrs/day)(3600s/hr)$

$W_{cyc} = (3.33 \times 10^2\ \frac{MJ}{s})(48wks)(7\ \frac{days}{wk})(24\ hrs/day)(3.6 \times 10^3\ \frac{s}{hr})$

$W_{cyc} = (9.67 \times 10^4)(10^2)(10^3)\ MJ$

$\boxed{W_{cyc} = 9.67 \times 10^9\ MJ}$ ← ANS.

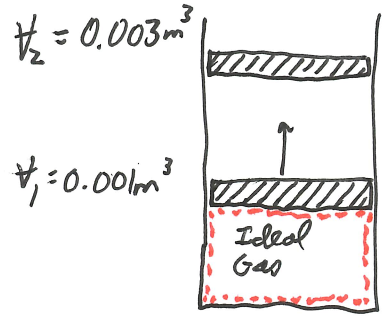
2. (10 points) Consider an ideal gas contained in a piston cylinder assembly. The gas expands from 0.001 m^3 to 0.003 m^3 in an isothermal process at 27°C. Compute the entropy produced in units of kJ/kg K. Note that $\ln(1/x) = -\ln(x)$

GIVEN: Ideal gas; V_1, V_2; Isothermal process, $T = 27°C$

FIND: $\sigma = ?$ kJ/kg·K

ASSUME: $T_{boundary} = T_{air} = 27°C = 300K$; No KE or PE effects

DIAGRAM



ANALYSIS: $\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma$

$$m(s_2 - s_1) = \frac{1}{T}\int \delta Q + \sigma$$

$$s_2 - s_1 = \frac{Q}{mT} + \frac{\sigma}{m} \Rightarrow \boxed{\frac{\sigma}{m} = s_2 - s_1 - \frac{Q}{mT}}$$

$$\Delta E = Q - W \qquad \Delta U = Q - W \rightarrow m(u_2 - u_1) = Q - W$$

b/c no KE or PE effects

For an ideal gas, $u = u(T)$, so $\Delta u = 0$ $\qquad Q - W = 0 \qquad Q = W \qquad \frac{Q}{m} = \frac{W}{m}$

$$\boxed{\frac{\sigma}{m} = s_2 - s_1 - \frac{W}{mT}}$$

$$W = m\int p\,dv \qquad pv = RT \qquad p = \frac{RT}{v} \qquad W = m\int_1^2 \frac{RT}{v}\,dv = mRT\int_1^2 dv/v$$

b/c isothermal

$$W = mRT\ln\left(\frac{v_2}{v_1}\right) \qquad \frac{W}{m} = RT\ln\left(\frac{v_2}{v_1}\right) = \boxed{RT\ln(3) = \frac{W}{m}}$$

For an ideal gas, $s_2 - s_1 = \cancel{s^\circ(T_2) - s^\circ(T_1)} - R\ln\left(\frac{P_2}{P_1}\right)$

0 b/c isothermal

$$\frac{\sigma}{m} = -R\ln\left(\frac{P_2}{P_1}\right) - \frac{R\cancel{T}}{\cancel{T}}\ln(3)$$

$$p = \frac{RT}{v}, \text{ so } \frac{P_2}{P_1} = \frac{RT/v_2}{RT/v_1} = \frac{v_1}{v_2}$$

$$\frac{\sigma}{m} = -R\ln\left(\frac{v_1}{v_2}\right) - R\ln(3)$$

$$\frac{\sigma}{m} = -R\left[\ln\left(\frac{1}{3}\right) + \ln(3)\right]$$

since $\ln\left(\frac{1}{x}\right) = -\ln(x) \rightarrow \frac{\sigma}{m} = -R[-\ln(3) + \ln(3)]$

$$\boxed{\frac{\sigma}{m} = 0} \leftarrow \text{ANS.}$$

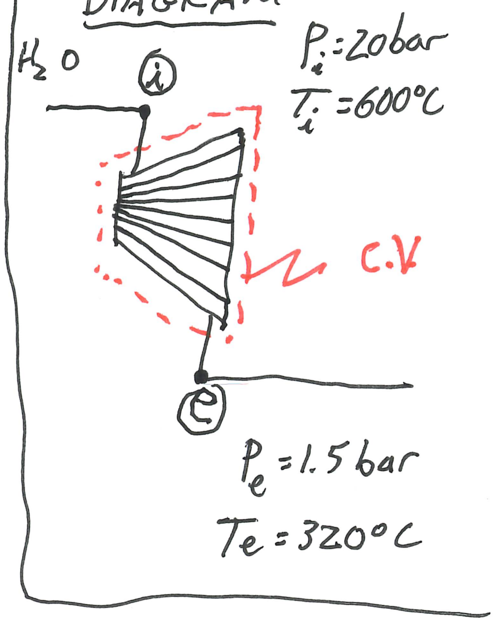
3. (10 points) Steam enters a turbine at 20 bar and 600°C and exits at 1.5 bar and 320°C. Compute the isentropic efficiency of this turbine.

GIVEN: H_2O, P_i, T_i, P_e, T_e

FIND: η_t

ASSUME: Assumptions used in isentropic efficiency computations

DIAGRAM



ANALYSIS

$$\boxed{\eta_t = \frac{\dot{W}/\dot{m}}{(\dot{W}/\dot{m})_s}}$$

For isentropic efficiencies, we assume S.S., no KE or PE effects, adiabatic turbine, so $\dot{W}/\dot{m} = h_i - h_e$

$$\eta_t = \frac{h_i - h_e}{h_i - h_{e,s}}$$

From table,

$$\boxed{\begin{aligned} h_i &= 3690.1 \ \tfrac{kJ}{kg} \\ s_i &= 7.7024 \ kJ/kg \cdot K \\ h_e &= 3113.5 \ \tfrac{kJ}{kg} \end{aligned}}$$

For $h_{e,s}$ we set $s_e = s_i = 7.7024 \frac{kJ}{kg \cdot K}$ and interpolate for h at 1.5 bar

p = 1.5 bar

$h\left(\frac{kJ}{kg}\right)$	$s\,(kJ/kg \cdot K)$
2872.9	7.6433
($h_{e,s}$)	(7.7024)
2952.7	7.8052

$$\frac{(7.8052 - 7.6433)(kJ/kg \cdot K)}{(2952.7 - 2872.9)\ kJ/kg} = \frac{(7.7024 - 7.6433)\frac{kJ}{kg \cdot K}}{(h_{e,s} - 2872.9)\frac{kJ}{kg}}$$

$$\boxed{h_{e,s} = 2902.0 \ \frac{kJ}{kg}}$$

$$\eta_t = \frac{(3690.1 - 3113.5)\ kJ/kg}{(3690.1 - 2902.0)\ kJ/kg} = \frac{576.6 \ kJ/kg}{788.1 \ kJ/kg}$$

$$\boxed{\eta_t = 0.7316}$$ ← ANS.

4. (5 points) Derive an equation for Δs for an ideal gas with constant specific heats, using the first Tds equation in the equation sheet.

GIVEN: Tds equation

FIND: Equation for Δs

ASSUME: Ideal gas behavior; constant (c_p, c_v)

ANALYSIS: $Tds = du + pdv$

$$ds = \frac{du}{T} + \frac{p}{T}dv$$

Since c_v = constant, we can write $c_v = \frac{du}{dT} \rightarrow du = c_v dT$

Since we can assume ideal gas behavior: $pv = RT$ $p = \frac{RT}{v}$

$$\int_1^2 ds = \int_1^2 c_v \frac{dT}{T} + \int_1^2 R \frac{\cancel{T}}{\cancel{T}} \frac{dv}{v}$$

b/c constant

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$\boxed{\Delta s = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)} \leftarrow \underline{\text{ANS.}}$$

I HAVE NEITHER PROVIDED OR RECEIVED HELP DURING THIS EXAM.

SIGNATURE