EXAM #1 – ME 3030 – Section 002 – Thermodynamics – Spring 2024
Prof. J. R. Saylor

NAME: SOLUTION

This is a closed book/closed notes exam. Use of a 4-function calculator is permitted. Zero credit will be earned for this exam if the honors pledge is not signed.

1. (10 points) A heat pump keeps a house at 25°C on a day when the outside temperature is −5°C. Because the house is not perfectly insulated, heat leaks out of the house at a rate of 2.3 kW. If the coefficient of performance for the heat pump is 54% of the coefficient of performance for a perfectly reversible heat pump, compute the power input for the actual heat pump in Watts.

GIVEN: T_c, T_H, $\dot{Q}_H$, $\gamma = 0.54\gamma_{max}$

FIND: $\dot{W}_{cyc} = ?W$

ASSUME:

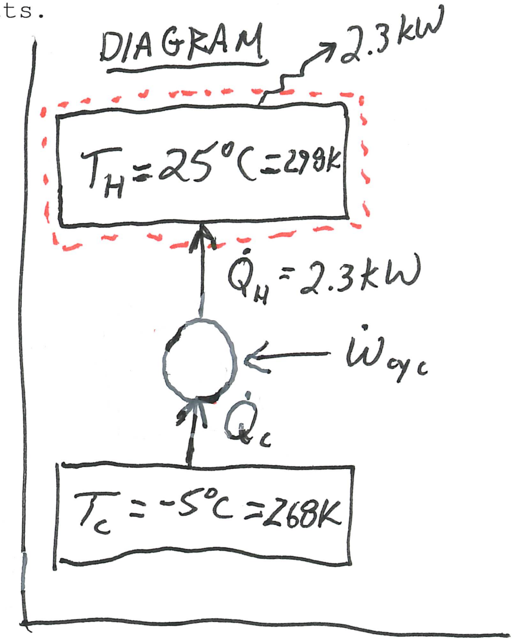
ANALYSIS: $\gamma_{max} = \frac{T_H}{T_H - T_C} = \frac{298K}{298K - 268K} = \boxed{9.933}$

$\gamma = 0.54\gamma_{max} = (0.54)(9.933) = \boxed{5.36}$

$\gamma = \frac{\dot{Q}_H}{\dot{W}_{cyc}} = \frac{2.3kW}{\dot{W}_{cyc}} = 5.36$

$\dot{W}_{cyc} = 0.429\ kW$

$\boxed{\dot{W}_{cyc} = 429W}$ ← ANS.



2. (5 points) Derive an equation for Δs for an ideal gas with constant specific heats, using the second Tds equation in the equation sheet.

GIVEN: Tds equation

FIND: Equation for Δs

ASSUME: Ideal gas behavior; constant (c_p, c_v)

ANALYSIS: $T ds = dh - v\,dp$

$$ds = \frac{dh}{T} - \frac{v}{T} dp$$

Since c_p = constant, we can write $c_p = \frac{dh}{dT}$ $\boxed{dh = c_p\, dT}$

Since we can assume ideal gas behavior:

$$pv = RT$$

$$\boxed{\frac{v}{T} = \frac{R}{P}}$$

Substituting:

$$ds = c_p \frac{dT}{T} - R \frac{dp}{P}$$

Integrating:

$$\int_1^2 ds = \int_1^2 c_p \frac{dT}{T} - \int_1^2 R \frac{dp}{P}$$

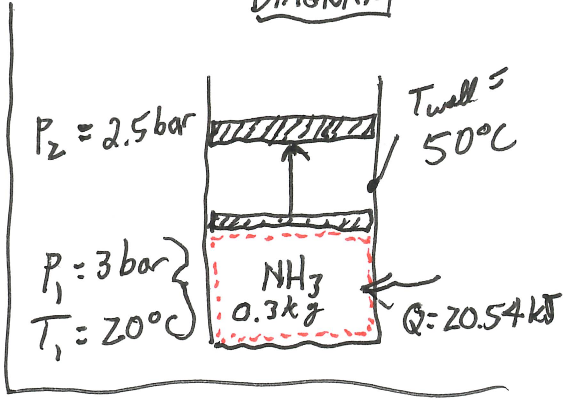
Pulling constants outside the integral

$$s_2 - s_1 = c_p \int_1^2 \frac{dT}{T} - R \int_1^2 \frac{dp}{P}$$

$$\boxed{\Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)}$$

3. (10 points) Ammonia is contained in a piston-cylinder assembly and is initially at 3 bar and 20°C. A heat transfer of 20.54 kJ to the ammonia occurs at the cylinder wall where the temperature is 50°C. The ammonia expands and the pressure drops to 2.5 bar. If the mass of the ammonia is 0.3 kg, what is the maximum possible work the ammonia can deliver to the environment?

DIAGRAM



GIVEN: NH_3, P_1, T_1, P_2, T_{wall}, Q, m

FIND: Max possible work?

ASSUME: No KE or PE effects

ANALYSIS: Closed system

$\Delta E = Q - W \rightarrow \Delta U = Q - W \rightarrow m(u_2 - u_1) = Q - W$

b/c no KE or PE effects

$$W = Q + m(u_1 - u_2)$$

$\Delta S = \int_1^2 \frac{\delta Q}{T} + \sigma \qquad m(s_2 - s_1) = \frac{1}{T_w}\int \delta Q + \sigma$

$\sigma \rightarrow 0$ ← b/c $\sigma = 0$ for max work

b/c T_w is constant and uniform

$m(s_2 - s_1) = \frac{Q}{T_w} \rightarrow s_2 - s_1 = \frac{Q}{mT_w} \rightarrow$

$$s_2 = s_1 + \frac{Q}{mT_w}$$

From NH_3 table

① $u_1 = 1364.13$ kJ/kg

$s_1 = 5.7103$ kJ/kg·K

$s_2 = 5.7103 \frac{kJ}{kg \cdot K} + \frac{20.54 \text{ kJ}}{(0.3 \text{ kg})(50+273) K}$ (50+273 = 323K)

$$s_2 = 5.9223 \frac{kJ}{kg \cdot K}$$

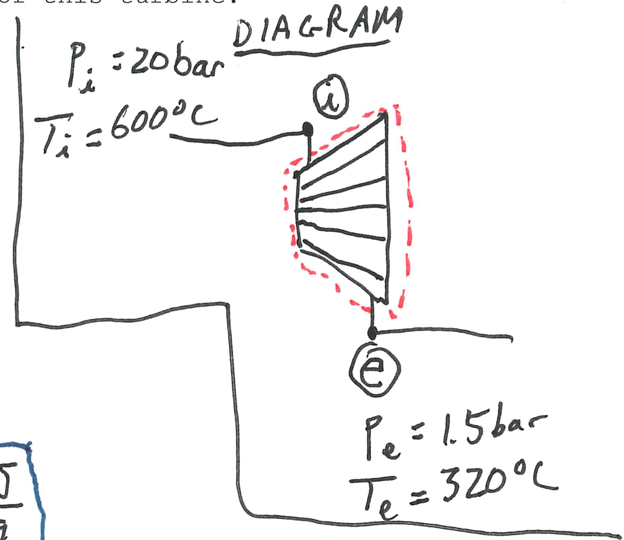
At p = 2.5 bar and s = 5.9223, interpolate for u_2... but value is exact, so interpolation not needed

$$u_2 = 1393.10 \text{ kJ/kg}$$ ← From NH_3 table.

$\rightarrow W = W_{max} = 20.54 \text{ kJ} + (0.3 \text{ kg})(1364.13 \frac{kJ}{kg} - 1393.10 \frac{kJ}{kg})$

$$W_{max} = 11.849 \text{ kJ}$$ ← ANS.

4. (10 points) Steam enters a turbine at 20 bar and 600°C and exits at 1.5 bar and 320°C. Compute the isentropic efficiency of this turbine.



GIVEN: P_i, T_i, P_e, T_e

FIND: η_t

ASSUME: Standard assumptions for η_t analysis: $\dot{W}/\dot{m} = h_i - h_e$

ANALYSIS:

$$\eta_t = \frac{\dot{W}/\dot{m}}{(\dot{W}/\dot{m})_s} = \frac{h_i - h_e}{h_i - h_{e,s}}$$

From H_2O Table

$h_i = 3690.1$ kJ/kg
$s_i = 7.7024$ kJ/kg

$h_e = 3113.5 \frac{kJ}{kg}$

$h_{e,s}$ is $h(1.5 \text{ bar}, s = s_i = 7.7024 \text{ kJ/kg})$

P = 1.5 bar

h (kJ/kg)	s (kJ/kg·K)
2872.9	7.6433
$h_{e,s}$	(7.7024)
2952.7	7.8052

$$\frac{7.8052 \frac{kJ}{kg \cdot K} - 7.6433 \frac{kJ}{kg \cdot K}}{2952.7 \frac{kJ}{kg} - 2872.9 \frac{kJ}{kg}} = \frac{7.7024 \frac{kJ}{kg \cdot K} - 7.6433 \frac{kJ}{kg \cdot K}}{h_{e,s} - 2872.9 \text{ kJ/kg}}$$

$$h_{e,s} = 2902.0 \frac{kJ}{kg}$$

$$\eta_t = \frac{\left(3690.1 \frac{kJ}{kg} - 3113.5 \frac{kJ}{kg}\right)}{\left(3690.1 \frac{kJ}{kg} - 2902.0 \frac{kJ}{kg}\right)}$$

$$\eta_t = 0.7316$$ ← ANS.

I HAVE NEITHER PROVIDED OR RECEIVED HELP DURING THIS EXAM.

SIGNATURE