11.51 $\quad Z = 1 + \frac{Bp}{RT} \; ; \; B = B(T)$

<u>Find</u> $[h(P_2,T) - h(P_1,T)]$, $[u(P_2,T) - u(P_1,T)]$, $[s(P_2,T) - s(P_1,T)]$

<u>Note</u>: This is an isothermal process

Eq. (11.60) $\quad h_2 - h_1 = \int_1^2 \underbrace{\cancel{c_p dT}}_{0} + \int_1^2 \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp$

$$Z = \frac{pv}{RT} = 1 + \frac{Bp}{RT}$$

$$v = \frac{RT}{p} + B$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{dB}{dT}$$

$$h_2 - h_1 = \int_1^2 \left[ \cancel{\frac{RT}{p}} + B - T\left(\cancel{\frac{R}{p}} + \frac{dB}{dT}\right) \right] dp$$

$$\boxed{h_2 - h_1 = \underbrace{\left[ B - \frac{dB}{dT} \right]} (P_2 - P_1)}$$

<u>Note</u>: This term is only a function of T

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Eq. (11.51) $\quad u_2 - u_1 = \int_1^2 \underbrace{\cancel{c_v dT}}_{0} + \int_1^2 \left[ T\left(\frac{\partial p}{\partial T}\right)_v - p \right] dv$

$$\frac{pv}{RT} = 1 + \frac{Bp}{RT} \Rightarrow P\left(\frac{v}{RT} - \frac{B}{RT}\right) = 1$$

$$\boxed{P = \frac{RT}{v - B}}$$

11.51 continued

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{(v-B)R - RT\left(\frac{-dB}{dT}\right)}{(v-B)^2} = \frac{R}{v-B} + \frac{RT}{(v-B)^2}\frac{dB}{dT}$$

$$u_2-u_1 = \int_1^2 \left[\cancel{\frac{RT}{v-B}} + \frac{RT^2}{(v-B)^2}\frac{dB}{dT} - \cancel{\frac{RT}{v-B}}\right] dv$$

$$= \int_1^2 \left(\frac{RT^2}{(v-B)^2}\frac{dB}{dT}\right) dv$$

$$= RT^2\frac{dB}{dT}\int_1^2 \frac{dv}{(v-B)^2}\, dv$$

$$u = v - B$$
$$du = dv$$

$$= RT^2\frac{dB}{dT}\left.\frac{(v-B)^{-1}}{-1}\right|_1^2$$

$$= \frac{-RT^2\frac{dB}{dT}}{v-B} = \left. -Tp\frac{dB}{dT}\right|_1^2$$

$$\boxed{u_2-u_1 = \cancel{-pT}\ -T\frac{dB}{dT}(p_2-p_1)}$$

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11.51 (continued)

$$s_2 - s_1 = \int_1^2 \cancelto{0}{\frac{c_p}{T}\,dT} - \int_1^2 \left(\frac{\partial v}{\partial T}\right)_p dP$$

$$s_2 - s_1 = -\int_1^2 \left(\frac{R}{P} + \frac{dB}{dT}\right) dP$$

$$\boxed{s_2 - s_1 = -\left[R \ln \frac{P_2}{P_1} + \frac{dB}{dT}(P_2 - P_1)\right]}$$