PROBLEM 11.55

KNOWN: Three cases are under consideration: (a) an ideal gas, (b) a gas whose equation of state is $p(v-b) = RT$, (c) a gas obeying the van der Waals equation.

FIND: Derive expressions for β and κ for each case.

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \quad \text{and} \quad \kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

(a) Ideal Gas: $v = RT/p$.

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{v}\left[\frac{R}{p}\right] = \frac{1}{T} \qquad \text{(a)}$$

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2}\right] = \frac{1}{p}\left[\frac{RT}{pv}\right] = \frac{1}{p}$$

(b) $v = (RT/p) + b$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}, \quad \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{v}\left[\frac{R}{p}\right] = \frac{R}{v}\left[\frac{v-b}{RT}\right] = \frac{1}{T}\left[\frac{v-b}{v}\right] \qquad \text{(b)}$$

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2}\right] = \frac{1}{p}\left[\frac{RT}{pv}\right] = \frac{1}{p}\left[\frac{RT/v}{RT/(v-b)}\right] = \frac{1}{p}\left[\frac{v-b}{v}\right]$$

Note: when $b=0$ these expressions reduce to those of part (a).

(c) van der Waals: $p = \frac{RT}{(v-b)} - \frac{a}{v^2}$.

As the van der Waals equation is not explicit in v, the required partial derivatives are not so easily found as in parts (a) and (b). Thus, following the procedure explained in Example 11.2, an expression for $(\partial v/\partial T)_p$ is obtained as

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{-R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]}$$

So

$$\beta = \frac{-R(v-b)v^2}{2a(v-b)^2 - RTv^3}$$

Using Eqs. 11.15 and 11.16

$$\left(\frac{\partial v}{\partial p}\right)_T = \frac{-(\partial v/\partial T)_p}{(\partial p/\partial T)_v} = \frac{\frac{R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]}}{R/(v-b)} = \frac{1}{\left[2a/v^3 - RT/(v-b)^2\right]} \qquad \text{(c)}$$

Thus

$$\kappa = -\frac{1}{v\left[2a/v^3 - RT(v-b)^2\right]} = \frac{-v^2(v-b)^2}{\left[2a(v-b)^2 - RTv^3\right]}$$