PROBLEM 13.79

KNOWN: A gaseous mixture of C_4H_{10} and 80% excess air enters a coolant-jacketed reactor at 25°C, 3atm. Complete combustion occurs and products exit at 1200 K, 3atm. Saturated liquid coolant enters the jacket and saturated vapor exits.

FIND: Determine (a) the mass flow rate of the coolant, in kg per kmol of fuel, (b) the rate of entropy production, (c) and the rate of exergy destruction. Consider two coolants: water at 1 bar and ammonia at 10 bar.

SCHEMATIC & GIVEN DATA:

$T_0 = 25°C$

Coolants:
• water at 1 bar
• ammonia at 10 bar

{C_4H_{10}, Air}
25°C, 3atm

{Products}
1200K, 3atm
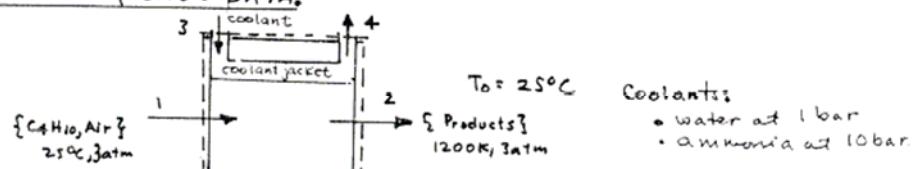
ENGINEERING
MODEL: (1) The control volume shown in the accompany figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The incoming fuel-air mixture and exiting combustion products can be modeled as ideal gases.

ANALYSIS: The complete combustion of C_4H_{10} with the theoretical amount of air is
$$C_4H_{10} + 6.5(O_2 + 3.76 N_2) \rightarrow 4 CO_2 + 5 H_2O + 24.44 N_2$$
Complete combustion with 80% excess air is then
$$C_4H_{10} + 11.7(O_2 + 3.76 N_2) \rightarrow 4 CO_2 + 5 H_2O + 5.2 O_2 + 43.992 N_2$$
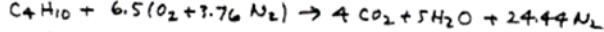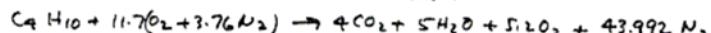
(a) An energy rate balance at steady reduces to the form
$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_{cw}[h_3 - h_4] + \dot{n}_{C_4H_{10}}\left[(\bar{h}_{C_4H_{10}} + 11.7\bar{h}_{O_2} + 43.992\bar{h}_{N_2})_1 - (4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} + 5.2\bar{h}_{O_2} + 43.992\bar{h}_{N_2})_2\right]$$

where $\dot{m}_{cw}$ is the mass flow rate of the coolant and $\dot{n}_{C_4H_{10}}$ is the molar flow rate of the C_4H_{10}. Accordingly

$$\frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{(\bar{h}_{C_4H_{10}} + 11.7\bar{h}_{O_2} + 43.992\bar{h}_{N_2})_1 - (4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} + 5.2\bar{h}_{O_2} + 43.992\bar{h}_{N_2})_2}{h_4 - h_3}$$

With $\bar{h} = \bar{h}_f^\circ + \delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{(\bar{h}_f^\circ)_{C_4H_{10}} - \{4[\bar{h}_f^\circ + \bar{h}(1200) - \bar{h}(298)]_{CO_2} + 5[\bar{h}_f^\circ + \bar{h}(1200) - \bar{h}(298)]_{H_2O} + 5.2[\bar{h}(1200) - \bar{h}(298)]_{O_2} + 43.992[\bar{h}(1200) - \bar{h}(298)]_{N_2}\}}{h_g - h_f}$$

$$= \frac{-126,150 - 4[-393,520 + 53,848 - 9364] - 5[-241,820 + 44,380 - 9904] - 5.2[38,447 - 8682] - 43.992[36,777 - 8669]}{h_g - h_f}$$

$$= \frac{915,409 \text{ kJ/kmol (fuel)}}{h_g - h_f} \qquad (1)$$

• For water at 1bar, $h_{fg} = 2258 \frac{kJ}{kg}$ ⟹ $\dfrac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \dfrac{915,409}{2258} = 405.4 \dfrac{kg}{kmol(fuel)}$ ⟵ (a)

• For ammonia at 10 bar, $h_{fg} = 1165.4 \frac{kJ}{kg}$ ⟹ $\dfrac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = 785.5 \dfrac{kg}{kmol(fuel)}$

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Problem 13-79 continued

(b) An entropy rate balance at steady state reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = (4\,\bar{s}_{CO_2} + 5\,\bar{s}_{H_2O} + 5.2\,\bar{s}_{O_2} + 43.992\,\bar{s}_{N_2}) - (\bar{s}_{C_4H_{10}} + 11.7\,\bar{s}_{O_2} + 43.992\,\bar{s}_{N_2}) + \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}}(s_4 - s_3) \quad (1)$$

The fuel and air enter as a mixture at 25°C, 3 atm with the composition,
$y_{C_4H_{10}} = 1/56.692$, $y_{O_2} = 11.7/56.692$, $y_{N_2} = 43.992/56.692$. Accordingly

$$\bar{s}_{C_4H_{10}} = \bar{s}^{\circ}_{C_4H_{10}}(298) - \bar{R}\ln\frac{y_{C_4H_{10}}P}{P_{ref}} = 310.03 - 8.314\ln\frac{3}{56.692} = 334.46 \text{ KJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(298) - \bar{R}\ln\frac{y_{O_2}P}{P_{ref}} = 205.03 - 8.314\ln\frac{(11.7)(3)}{56.692} = 209.02 \text{ KJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(298) - \bar{R}\ln\frac{y_{N_2}P}{P_{ref}} = 191.5 - 8.314\ln\frac{(43.992)(3)}{56.692} = 184.47 \text{ KJ/kmol·K}$$

The products exit as a mixture at 1200K, 3 atm with the composition,
$y_{CO_2} = 4/58.192$, $y_{H_2O} = 5/58.192$, $y_{O_2} = 5.2/58.192$, $y_{N_2} = 43.992/58.192$. Accordingly

$$\bar{s}_{CO_2} = \bar{s}^{\circ}_{CO_2}(1200) - \bar{R}\ln\frac{y_{CO_2}P}{P_{ref}} = 279.307 - 8.314\ln\frac{(4)(3)}{58.192} = 292.43 \text{ KJ/kmol·K}$$

$$\bar{s}_{H_2O} = \bar{s}^{\circ}_{H_2O}(1200) - \bar{R}\ln\frac{y_{H_2O}P}{P_{ref}} = 240.333 - 8.314\ln\frac{(5)(3)}{58.192} = 251.60 \text{ KJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(1200) - \bar{R}\ln\frac{y_{O_2}P}{P_{ref}} = 249.906 - 8.314\ln\frac{(5.2)(3)}{58.192} = 260.85 \text{ KJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(1200) - \bar{R}\ln\frac{y_{N_2}P}{P_{ref}} = 234.115 - 8.314\ln\frac{(43.992)(3)}{58.192} = 227.31 \text{ KJ/kmol·K}$$

From Table A-3, $s_4 - s_3 = (7.3559 - 1.306) = 6.0499 \text{ KJ/kg·K}$.

Substituting values into Eq.(1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 4(292.43) + 5(251.60) + 5.2(260.85) + 43.992(227.31) - 334.46 - 11.7(209.02) - 43.992(184.47)$$
$$+ \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}(s_4 - s_3)$$

$$= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}(s_4 - s_3)$$

$$= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}s_{fg}$$

The value for s_{fg} can be obtained directly from table data, but it is more instructive to rewrite the last expression using $s_{fg} = h_{fg}/T_{sat}$ from Sec. 6.3. Then

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}\cdot\frac{h_{fg}}{T_{sat}} = 2888.77 + \left[\frac{915,409}{h_{fg}}\right]\left[\frac{h_{fg}}{T_{sat}}\right]$$
$$\underbrace{}_{\text{Eq.(1)}}$$

$$= 2888.77 + \frac{915,409}{T_{sat}} \quad (2)$$

- Water at 1bar, $T_{sat} = 373K \Rightarrow \dfrac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \dfrac{915,409}{373} = 5342.9 \dfrac{\text{KJ}}{\text{Kmol·K}}$

① (b)

- Ammonia at 10bar, $T_{sat} = 298K \Rightarrow \dfrac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \dfrac{915,409}{298} = 5960.6 \dfrac{\text{KJ}}{\text{Kmol·K}}$

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Problem 13-79 continued

(c) The exergy destruction is obtained using

$$\frac{\dot{E}_d}{\dot{n}_{C_4 H_{10}}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4 H_{10}}}$$

- water at 1 bar:

$$\frac{\dot{E}_d}{\dot{n}_{C_4 H_{10}}} = (298K)(5342.9) = 1.592 \times 10^6 \frac{KJ}{kmol\,(fuel)}$$

(c)

- ammonia at 10 bars:

$$\frac{\dot{E}_d}{\dot{n}_{C_4 H_{10}}} = (298)(5960.6) = 1.776 \times 10^6 \frac{KJ}{kmol\,(fuel)}$$

1. The difference between the entropy production rates is not due to the natures of the two coolants, but only to the temperature T_{sat}, as shown by Eq. (?). If each coolant were at the same temperature, the same entropy production rate would be determined for each. The choice of the coolant is determined by other considerations. For example, at a temperature of $T_{sat} = 50°C$ the pressure of saturated water is just 0.1235 bar, whereas the pressure of saturated ammonia is 20.331 bar. Such extreme pressures are likely to be avoided in applications of the type under consideration, however. And as shown by part(a) the temperature T_{sat} also affects the mass flow rate.