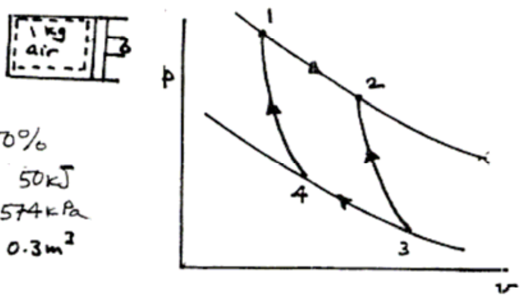
PROBLEM 5.77

KNOWN: One kg of air undergoes a Carnot cycle for which $\eta = 50\%$.

FIND: Determine the minimum and maximum temperatures, the pressure and volume at the beginning of the isothermal expansion, the work and heat transfer for each process, and sketch the cycle on p-v coordinates.

SCHEMATIC & GIVEN DATA:



$\eta = 50\%$

$Q_{12} = 50 kJ$

$P_2 = 574 kPa$

$V_2 = 0.3 m^3$

Summary:

process	Q	W
1-2	50	50
2-3	0	220.7
3-4	-25	-25
4-1	0	-220.7
cycle	25	25

ENGR. MODEL: 1. The system shown in the schematic consists of air modeled as an ideal gas. 2. Volume change is the only work mode.

ANALYSIS: (a) Using the ideal gas model equation of state, $T_2 = P_2 V_2 / mR$. Then

$$T_2 = \frac{(574 \times 10^3 \frac{N}{m^2})(0.3 m^3)}{\left(\frac{8314}{28.97} \frac{N\cdot m}{kg\cdot K}\right)(1 kg)} = 600 K$$ Then, since $\eta = 1 - \frac{T_C}{T_H} \Rightarrow T_C = T_H(1-\eta)$. With $T_H = T_2$, ◄ (a)

$T_C = 600(1-0.5) = 300 K$, where $T_C = T_3 = T_4$.

(b) For process 1-2, $Q_{12} = 50 kJ$ (given). An energy balance reads $m(u_2-u_1) = Q_{12} - W_{12}$, but since internal energy of an ideal gas depends on temperature and $T_1 = T_2$, $W_{12} = Q_{12}$. Further, $W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT_H}{V} dV = mRT_H \ln V_2/V_1$. Solving and inserting values

$$\ln \frac{V_2}{V_1} = \frac{W_{12}}{mRT_H} \Rightarrow \ln \frac{V_2}{V_1} = \frac{50 kJ}{(1 kg)\left(\frac{8.314}{28.97} \frac{kJ}{kg\cdot K}\right)(600 K)} = 0.2904 \Rightarrow V_1 = 0.224 m^3$$ ◄ (b)

Since $T_1 = T_2$, $P_1 V_1 = mRT$, $P_2 V_2 = mRT \Rightarrow P_2 V_2 = P_1 V_1$, $P_1 = P_2(V_2/V_1) = 574 kPa(0.3/0.224) = 769 kPa$.

(c) For process 2-3: $Q_{23} = 0$. An energy balance reduces to give $W_{23} = m(u_2 - u_3)$ with data from Table A-22, $W_{23} = (1 kg)(434.78 - 214.07) = 220.7 kJ$. For process 3-4, $W_{34} = Q_{34}$ (as for process 1-2). Also, Eq. 5.7 is applicable: ◄ (c)

$$\frac{|Q_{34}|}{T_C} = \frac{Q_{12}}{T_H} \Rightarrow |Q_{34}| = \left(\frac{300}{600}\right)(50 kJ) = 25 kJ. \Rightarrow Q_{34} = -25 kJ, W_{34} = -25 kJ.$$

① Process 4-1, $Q_{41} = 0$. An energy balance reduces to give, $W_{41} = m(u_4 - u_1)$. Since $u_1 = u_2$, $u_4 = u_3$, $W_{41} = -220.7 kJ$. The work and heat transfers are summarized in the table.

1. As checks on the calculations, note that (1) $W_{cycle} = Q_{cycle}$ (see summary above), and (2)

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{25}{50} = 0.5$$