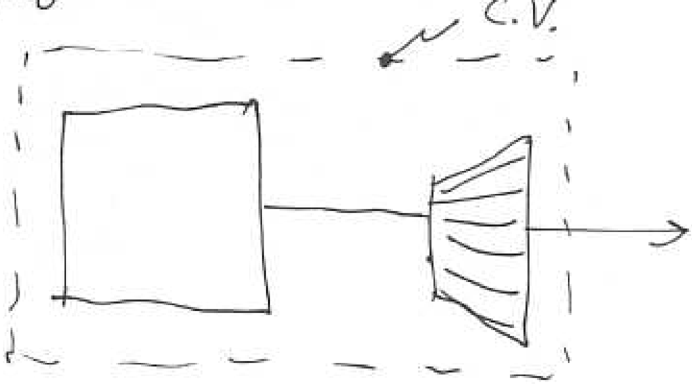
6.138 GIVEN Air Ideal Gas, $\dot{Q} = 0$

$\Delta KE = \Delta PE = 0$

Want: Max theoretical work $\Rightarrow \sigma = 0$

$\dot{\sigma} = 0$



(1)	(2)
$P_1 = 0.5\ \text{MPa}$	$P_2 = 0.1\ \text{MPa}$
$T_1 = 500\text{K}$	$T_2 = ?$
$m_1 = 5\text{kg}$	$m_2 = ?$

First, C.O.E.

$$\frac{dE}{dt} = \cancel{\dot{Q}} - \dot{W} + \cancel{\dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)}^{\,0 \text{ no inlet}} - \dot{m}_e\left(h_e + \cancel{\frac{V_e^2}{2}} + \cancel{gz_e}\right)$$

$\Delta KE = \Delta PE = 0$

$$\frac{dU}{dt} = -\dot{W} - \dot{m}_e h_e \Rightarrow \int dU = \int_1^2 -\dot{W}\,dt - \int_1^2 \dot{m}_e h_e\,dt$$

$$U_2 - U_1 = -W - h_e m_e \qquad m_e = m_1 - m$$

$$m_2 u_2 - m_1 u_1 = -W - h_e(m_1 - m_2)$$

$$\boxed{W = m_1 u_1 - m_2 u_2 + h_e(m_2 - m_1)} \text{ — } (A)$$

Note $\quad pV = mRT \qquad V = \frac{mRT}{p} \qquad V_1 = V_2 \Rightarrow \frac{m_1 \cancel{R} T_1}{P_1} = \frac{m_2 \cancel{R} T_2}{P_2}$

$$\frac{m_2}{m_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right)$$

Once we get T_2, we can get u_2, m_2 . . .

6.138 (continued)

As the air exits the tank and goes through the turbine, does s change? For the moment lets follow a fixed parcel of mass (air) as it moves through both the tank and turbine

Tank since we are assuming $\sigma = 0$, ~~then~~ and $Q = 0$

$$\Delta S = \int \frac{\delta Q}{T} + \sigma$$

$$\Delta S = \cancel{m\Delta s} \; m\Delta s$$

since we are looking at a fixed amount of mass, $\Delta s = 0$

Turbine $\Rightarrow$ same result $\Delta s = 0$

Hence the process is isentropic. For air as an ideal gas in an isentropic process, we know

$$\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1}$$

From Table A-22 $P_{r1} = 8.411$

$\therefore P_{r2} = 1.682$

Using A-22 again gives $\boxed{T_2 = 317K}$

Hence, $m_2 = (5.0kg)\left(\frac{0.1 MPa}{0.5MPa}\right)\left(\frac{500K}{317K}\right) = \underline{1.58 kg}$

Since the turbine exit is $\underline{\underline{at}}$ P_2, $T_e = T_2$

Hence
$$\left.\begin{aligned} u_2 &= u(T_2) \\ h_e &= h(T_2) \\ u_1 &= u(T_1) \end{aligned}\right\} \text{FROM Table A-22} \left\{\begin{aligned} &= 226.3 \text{ kJ/kg} \\ &= 317.3 \text{ kJ/kg} \\ &= 359.49 \text{ kJ/kg} \end{aligned}\right.$$

Using (A) $W = m_1 u_1 - m_2 u_2 + h_e(m_2 - m_1) = \boxed{354.7 kJ}$ ← Ans.