

Computational Results for One-List and Two-List Label-Correcting Shortest Path Algorithms

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1 Computational Results for One-List Algorithms

The following tables display various computational measures (number of node scans, number of comparisons, number of updates) obtained by running algorithms LIFO, FIFO, PAPE, and PRED on randomly generated directed networks. The generated networks have source node $s = 1$ and every node is guaranteed to be accessible from s . A total of 30 replications were obtained for each network having $n = 100, 150, 200, 250, 500, 750, 1000$ nodes and density $\delta = \frac{m}{n} = 3, 4, 5, 10, 15, 20, 25, 30$. Also, we tabulate (a) the number of times a locally nonsharp node is scanned by FIFO, (b) the number of node scans per number of nodes, and (c) the number of updates per number of node scans. In view of its excessive computation time, results for LIFO are not given for $n \geq 500$.

1.1 Results for $n = 100$

Tables 1–3 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms LIFO, FIFO, PAPE, and PRED on randomly generated networks with $n = 100$ nodes and various densities δ . Also, Table 4 shows the number of times a locally nonsharp node is scanned by FIFO, Table 5 shows the number of node scans per number of nodes, and Table 6 shows the number of updates per number of node scans.

Table 1: Number of Node Scans ($n = 100$)

Density	LIFO	FIFO	PAPE	PRED
3	1067.8	133.7	118.5	119.3
4	1438.3	145.5	133.0	132.5
5	1529.8	157.8	142.2	141.2
10	1739.7	182.8	177.1	165.0
15	1654.3	193.5	203.7	178.1
20	1477.5	202.6	229.1	189.5
25	1486.3	207.7	229.2	190.9
30	1187.5	205.5	237.0	192.8

Table 2: Number of Comparisons ($n = 100$)

Density	LIFO	FIFO	PAPE	PRED
3	3170.2	401.3	353.7	356.8
4	5718.2	581.5	533.5	532.4
5	7583.4	789.1	708.8	706.5
10	17443.0	1826.1	1768.7	1651.3
15	24891.0	2902.6	3049.2	2668.3
20	29494.0	4048.7	4569.6	3791.5
25	37072.0	5199.6	5740.2	4789.7
30	35663.0	6158.2	7095.4	5787.4

Table 3: Number of Updates ($n = 100$)

Density	LIFO	FIFO	PAPE	PRED
3	1071.1	159.1	152.6	152.1
4	1445.6	189.8	189.4	187.0
5	1544.4	218.1	216.7	214.1
10	1797.2	313.3	329.8	320.5
15	1768.6	368.4	399.4	386.1
20	1640.7	410.5	460.7	436.4
25	1707.5	444.2	499.2	475.4
30	1435.0	468.6	543.2	514.0

Table 4: Number of Times a Locally Nonsharp Node is Scanned ($n = 100$)

Density	FIFO
3	11.77
4	10.73
5	12.33
10	16.70
15	16.73
20	18.13
25	21.27
30	18.27

1.2 Results for $n = 150$

Tables 7–9 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms LIFO, FIFO, PAPE, and PRED on randomly generated networks with $n = 150$ nodes and various densities δ . Also, Table 10 shows the number of times a locally nonsharp node is scanned by FIFO, Table 11 shows the number of node scans per number of nodes, and Table 12 shows the number of updates per number of node scans.

Table 5: Number of Node Scans per Node ($n = 100$)

Density	LIFO	FIFO	PAPE	PRED
3	10.7	1.34	1.19	1.19
4	14.4	1.45	1.33	1.32
5	15.3	1.58	1.42	1.41
10	17.4	1.83	1.77	1.65
15	16.5	1.93	2.04	1.78
20	14.8	2.03	2.29	1.90
25	14.9	2.08	2.29	1.91
30	11.9	2.06	2.37	1.93

Table 6: Number of Updates per Scan ($n = 100$)

Density	LIFO	FIFO	PAPE	PRED
3	1.00	1.19	1.29	1.27
4	1.01	1.30	1.42	1.41
5	1.01	1.38	1.52	1.52
10	1.03	1.71	1.86	1.94
15	1.07	1.90	1.96	2.17
20	1.11	2.03	2.01	2.30
25	1.15	2.14	2.18	2.49
30	1.21	2.28	2.29	2.67

Table 7: Number of Node Scans ($n = 150$)

Density	LIFO	FIFO	PAPE	PRED
3	2603.6	212.3	185.4	189.7
4	3165.9	228.2	200.7	201.9
5	3635.9	235.8	215.9	213.7
10	4330.8	287.4	280.2	261.8
15	4075.6	302.9	327.1	281.5
20	4051.9	313.3	351.4	287.8
25	3436.3	313.5	340.8	291.1
30	3559.0	334.0	377.4	300.4

Table 8: Number of Comparisons ($n = 150$)

Density	LIFO	FIFO	PAPE	PRED
3	7798.4	636.4	557.6	570.6
4	12490.0	909.7	798.5	806.3
5	18186.0	1178.4	1072.9	1065.9
10	43230.0	2870.6	2805.3	2624.1
15	61151.0	4539.4	4925.5	4238.8
20	80934.0	6275.3	7074.4	5772.7
25	86075.0	7842.1	8533.0	7296.3
30	106820.0	10012.0	11332.0	9018.8

Table 9: Number of Updates ($n = 150$)

Density	LIFO	FIFO	PAPE	PRED
3	2608.6	253.1	240.8	241.3
4	3179.7	295.8	289.9	286.8
5	3655.7	330.0	336.5	330.8
10	4417.0	488.2	525.5	505.2
15	4250.2	571.5	644.9	607.8
20	4321.4	635.9	721.0	675.6
25	3767.3	662.9	741.2	709.1
30	3997.4	744.4	845.1	793.2

Table 10: Number of Times a Locally Nonsharp Node is Scanned ($n = 150$)

Density	FIFO
3	20.90
4	22.90
5	20.03
10	28.47
15	28.60
20	31.73
25	27.63
30	32.63

Table 11: Number of Node Scans per Node ($n = 150$)

Density	LIFO	FIFO	PAPE	PRED
3	17.36	1.42	1.24	1.26
4	21.11	1.52	1.34	1.35
5	24.24	1.57	1.44	1.42
10	28.87	1.92	1.87	1.75
15	27.17	2.02	2.18	1.88
20	27.01	2.09	2.34	1.92
25	22.91	2.09	2.27	1.94
30	23.73	2.23	2.52	2.00

Table 12: Number of Updates per Scan ($n = 150$)

Density	LIFO	FIFO	PAPE	PRED
3	1.00	1.19	1.30	1.27
4	1.00	1.30	1.44	1.42
5	1.01	1.40	1.56	1.55
10	1.02	1.70	1.88	1.93
15	1.04	1.89	1.97	2.16
20	1.07	2.03	2.05	2.35
25	1.10	2.11	2.18	2.44
30	1.12	2.23	2.24	2.64

Table 13: Number of Node Scans ($n = 200$)

Density	LIFO	FIFO	PAPE	PRED
3	4683.3	286.7	248.9	253.1
4	5915.5	305.4	275.7	276.5
5	6730.1	318.2	284.0	284.3
10	7237.2	385.3	389.8	350.5
15	7156.3	407.9	429.5	377.7
20	6631.1	437.6	491.7	403.0
25	7094.0	444.6	512.8	416.8
30	6482.9	445.8	526.5	416.3

Table 14: Number of Comparisons ($n = 200$)

Density	LIFO	FIFO	PAPE	PRED
3	13967.0	858.9	744.8	757.9
4	23662.0	1213.5	1098.2	1104.6
5	33720.0	1595.3	1422.7	1429.6
10	72361.0	3844.2	3892.2	3506.8
15	107480.0	6107.0	6449.0	5680.6
20	132480.0	8741.7	9812.0	8054.5
25	177390.0	11132.0	12856.0	10454.0
30	194850.0	13371.0	15767.0	12502.0

1.3 Results for $n = 200$

Tables 13–15 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms LIFO, FIFO, PAPE, and PRED on randomly generated networks with $n = 200$ nodes and various densities δ . Also, Table 16 shows the number of times a locally nonsharp node is scanned by FIFO, Table 17 shows the number of node scans per number of nodes, and Table 18 shows the number of updates per number of node scans.

Table 15: Number of Updates ($n = 200$)

Density	LIFO	FIFO	PAPE	PRED
3	4689.7	340.5	325.9	325.3
4	5927.7	397.7	397.5	394.3
5	6757.0	439.3	437.3	435.9
10	7348.4	660.3	720.7	685.8
15	7378.9	772.2	859.6	823.4
20	6964.1	869.0	991.7	922.9
25	7587.0	950.0	1080.1	1014.5
30	7110.0	1001.7	1161.7	1086.0

Table 16: Number of Times a Locally Nonsharp Node is Scanned ($n = 200$)

Density	FIFO
3	30.33
4	28.47
5	31.13
10	39.10
15	41.67
20	42.87
25	44.43
30	43.67

Table 17: Number of Node Scans per Node ($n = 200$)

Density	LIFO	FIFO	PAPE	PRED
3	23.42	1.43	1.24	1.27
4	29.58	1.53	1.38	1.38
5	33.65	1.59	1.42	1.42
10	36.19	1.93	1.95	1.75
15	35.78	2.04	2.15	1.89
20	33.16	2.19	2.46	2.01
25	35.47	2.22	2.56	2.08
30	32.41	2.23	2.63	2.08

Table 18: Number of Updates per Scan ($n = 200$)

Density	LIFO	FIFO	PAPE	PRED
3	1.00	1.19	1.31	1.28
4	1.00	1.30	1.44	1.43
5	1.00	1.38	1.54	1.53
10	1.02	1.71	1.85	1.96
15	1.03	1.89	2.00	2.18
20	1.05	1.99	2.02	2.29
25	1.07	2.14	2.11	2.43
30	1.10	2.25	2.21	2.61

Table 19: Number of Node Scans ($n = 250$)

Density	LIFO	FIFO	PAPE	PRED
3	7639.2	377.6	327.0	339.0
4	9268.7	382.4	343.5	343.3
5	10545.0	430.7	384.5	379.1
10	11798.0	495.2	494.1	447.0
15	11679.0	548.5	582.6	508.3
20	11525.0	566.4	647.9	532.1
25	10867.0	554.4	646.1	509.5
30	10351.0	563.4	673.7	533.6

Table 20: Number of Comparisons ($n = 250$)

Density	LIFO	FIFO	PAPE	PRED
3	22837.0	1128.7	977.7	1013.5
4	36945.0	1530.3	1371.4	1376.4
5	52464.0	2152.4	1921.6	1900.5
10	117770.0	4942.6	4943.4	4476.3
15	174810.0	8191.1	8673.4	7597.9
20	231080.0	11326.0	12952.0	10651.0
25	271450.0	13850.0	16155.0	12774.0
30	310180.0	16896.0	20211.0	16033.0

1.4 Results for $n = 250$

Tables 19–21 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms LIFO, FIFO, PAPE, and PRED on randomly generated networks with $n = 250$ nodes and various densities δ . Also, Table 22 shows the number of times a locally nonsharp node is scanned by FIFO, Table 23 shows the number of node scans per number of nodes, and Table 24 shows the number of updates per number of node scans.

Table 21: Number of Updates ($n = 250$)

Density	LIFO	FIFO	PAPE	PRED
3	7647.1	450.2	432.7	433.5
4	9291.6	499.3	497.8	490.7
5	10582.0	595.6	600.6	584.0
10	11942.0	850.0	930.6	885.2
15	11989.0	1035.0	1169.6	1102.2
20	11993.0	1150.1	1321.7	1231.9
25	11488.0	1192.0	1377.5	1283.9
30	11154.0	1261.6	1471.1	1378.9

Table 22: Number of Times a Locally Nonsharp Node is Scanned ($n = 250$)

Density	FIFO
3	37.70
4	37.83
5	42.83
10	48.37
15	54.63
20	57.20
25	56.83
30	53.83

Table 23: Number of Node Scans per Node ($n = 250$)

Density	LIFO	FIFO	PAPE	PRED
3	30.56	1.51	1.31	1.36
4	37.07	1.53	1.37	1.37
5	42.18	1.72	1.54	1.52
10	47.19	1.98	1.98	1.79
15	46.72	2.19	2.33	2.03
20	46.10	2.27	2.59	2.13
25	43.47	2.22	2.58	2.04
30	41.40	2.25	2.69	2.13

Table 24: Number of Updates per Scan ($n = 250$)

Density	LIFO	FIFO	PAPE	PRED
3	1.00	1.19	1.32	1.28
4	1.00	1.31	1.45	1.43
5	1.00	1.38	1.56	1.54
10	1.01	1.72	1.88	1.98
15	1.03	1.89	2.01	2.17
20	1.04	2.03	2.04	2.32
25	1.06	2.15	2.13	2.52
30	1.08	2.24	2.18	2.58

Table 25: Number of Node Scans ($n = 500$)

Density	FIFO	PAPE	PRED
3	782.1	655.7	680.3
4	824.6	722.5	731.4
5	888.7	806.3	786.4
10	1092.5	1109.5	979.5
15	1127.0	1212.6	1027.3
20	1207.9	1402.2	1121.8
25	1218.8	1461.5	1155.0
30	1267.4	1551.7	1182.9

Table 26: Number of Comparisons ($n = 500$)

Density	FIFO	PAPE	PRED
3	2341.2	1965.5	2044.0
4	3301.8	2893.3	2936.4
5	4426.3	4027.0	3938.7
10	10921.0	11081.0	9824.5
15	16905.0	18193.0	15466.0
20	24166.0	28047.0	22529.0
25	30465.0	36514.0	28924.0
30	38008.0	46640.0	35568.0

1.5 Results for $n = 500$

Tables 25–27 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO, PAPE, and PRED on randomly generated networks with $n = 500$ nodes and various densities δ . Also, Table 28 shows the number of times a locally nonsharp node is scanned by FIFO, Table 29 shows the number of node scans per number of nodes, and Table 30 shows the number of updates per number of node scans.

Table 27: Number of Updates ($n = 500$)

Density	FIFO	PAPE	PRED
3	931.9	877.0	877.2
4	1068.2	1048.9	1043.1
5	1242.4	1274.2	1231.2
10	1847.6	2038.2	1912.7
15	2147.9	2442.3	2281.9
20	2430.6	2907.5	2660.6
25	2617.6	3158.4	2900.8
30	2780.0	3361.7	3059.5

Table 28: Number of Times a Locally Nonsharp Node is Scanned ($n = 500$)

Density	FIFO
3	87.20
4	89.40
5	97.07
10	120.70
15	124.50
20	132.97
25	131.13
30	142.50

Table 29: Number of Node Scans per Node ($n = 500$)

Density	FIFO	PAPE	PRED
3	1.56	1.31	1.36
4	1.65	1.45	1.46
5	1.78	1.61	1.57
10	2.19	2.22	1.96
15	2.25	2.43	2.05
20	2.42	2.80	2.24
25	2.44	2.92	2.31
30	2.53	3.10	2.37

Table 30: Number of Updates per Scan ($n = 500$)

Density	FIFO	PAPE	PRED
3	1.19	1.34	1.29
4	1.30	1.45	1.43
5	1.40	1.58	1.57
10	1.69	1.84	1.95
15	1.91	2.01	2.22
20	2.01	2.07	2.37
25	2.15	2.16	2.51
30	2.19	2.17	2.59

Table 31: Number of Node Scans ($n = 750$)

Density	FIFO	PAPE	PRED
3	1180.3	993.7	1029.6
4	1259.2	1109.5	1117.0
5	1408.9	1267.2	1239.2
10	1628.9	1631.1	1463.1
15	1788.6	1938.2	1626.9
20	1854.7	2094.3	1711.0
25	1932.3	2301.6	1785.7
30	1941.9	2356.8	1815.7

Table 32: Number of Comparisons ($n = 750$)

Density	FIFO	PAPE	PRED
3	3543.6	2984.7	3092.4
4	5030.3	4432.0	4475.4
5	7057.7	6353.0	6227.2
10	16273.0	16260.0	14646.0
15	26837.0	29107.0	24475.0
20	37080.0	41871.0	34310.0
25	48300.0	57539.0	44730.0
30	58283.0	70747.0	54626.0

1.6 Results for $n = 750$

Tables 31–33 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO, PAPE, and PRED on randomly generated networks with $n = 750$ nodes and various densities δ . Also, Table 34 shows the number of times a locally nonsharp node is scanned by FIFO, Table 35 shows the number of node scans per number of nodes, and Table 36 shows the number of updates per number of node scans.

Table 33: Number of Updates ($n = 750$)

Density	FIFO	PAPE	PRED
3	1412.1	1323.4	1329.6
4	1637.8	1625.2	1597.8
5	1957.9	2004.9	1938.7
10	2744.2	3029.3	2856.0
15	3376.1	3892.9	3609.9
20	3758.1	4381.2	4070.5
25	4089.7	4931.0	4495.4
30	4304.6	5206.8	4763.3

Table 34: Number of Times a Locally Nonsharp Node is Scanned ($n = 750$)

Density	FIFO
3	140.03
4	143.80
5	164.53
10	168.30
15	203.23
20	199.67
25	225.93
30	209.43

Table 35: Number of Node Scans per Node ($n = 750$)

Density	FIFO	PAPE	PRED
3	1.57	1.32	1.37
4	1.68	1.48	1.49
5	1.88	1.69	1.65
10	2.17	2.17	1.95
15	2.38	2.58	2.17
20	2.47	2.79	2.28
25	2.58	3.07	2.38
30	2.59	3.14	2.42

Table 36: Number of Updates per Scan ($n = 750$)

Density	FIFO	PAPE	PRED
3	1.20	1.33	1.29
4	1.30	1.46	1.43
5	1.39	1.58	1.56
10	1.68	1.86	1.95
15	1.89	2.01	2.22
20	2.03	2.09	2.38
25	2.12	2.14	2.52
30	2.22	2.21	2.62

Table 37: Number of Node Scans ($n = 1000$)

Density	FIFO	PAPE	PRED
3	1625.4	1378.7	1412.4
4	1752.1	1524.9	1543.1
5	1842.6	1658.2	1618.5
10	2249.7	2261.7	1995.9
15	2435.4	2635.4	2187.8
20	2567.5	2930.5	2343.0
25	2622.4	3102.8	2397.2
30	2629.8	3186.9	2424.1

Table 38: Number of Comparisons ($n = 1000$)

Density	FIFO	PAPE	PRED
3	4878.6	4133.6	4247.1
4	7013.4	6078.6	6182.9
5	9204.7	8273.8	8101.5
10	22496.0	22598.0	20010.0
15	36508.0	39526.0	32903.0
20	51365.0	58645.0	46989.0
25	65599.0	77649.0	60111.0
30	78916.0	95594.0	72866.0

1.7 Results for $n = 1000$

Tables 37–39 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO, PAPE, and PRED on randomly generated networks with $n = 1000$ nodes and various densities δ . Also, Table 40 shows the number of times a locally nonsharp node is scanned by FIFO, Table 41 shows the number of node scans per number of nodes, and Table 42 shows the number of updates per number of node scans.

Table 39: Number of Updates ($n = 1000$)

Density	FIFO	PAPE	PRED
3	1947.1	1850.6	1833.1
4	2277.0	2246.5	2202.3
5	2559.5	2612.9	2519.5
10	3777.8	4163.7	3893.2
15	4550.9	5222.6	4823.5
20	5231.2	6234.7	5725.6
25	5602.2	6760.9	6135.7
30	5831.8	7120.9	6474.8

Table 40: Number of Times a Locally Nonsharp Node is Scanned ($n = 1000$)

Density	FIFO
3	201.83
4	198.40
5	208.33
10	255.10
15	271.93
20	295.43
25	309.87
30	298.90

Table 41: Number of Node Scans per Node ($n = 1000$)

Density	FIFO	PAPE	PRED
3	1.63	1.38	1.41
4	1.75	1.52	1.54
5	1.84	1.66	1.62
10	2.25	2.26	2.00
15	2.44	2.64	2.19
20	2.57	2.93	2.34
25	2.62	3.10	2.40
30	2.63	3.19	2.42

Table 42: Number of Updates per Scan ($n = 1000$)

Density	FIFO	PAPE	PRED
3	1.20	1.34	1.30
4	1.30	1.47	1.43
5	1.39	1.58	1.56
10	1.68	1.84	1.95
15	1.87	1.98	2.20
20	2.04	2.13	2.44
25	2.14	2.18	2.56
30	2.22	2.23	2.67

Table 43: Number of Node Scans ($n = 100$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	133.7	124.9	124.8	124.6
4	145.5	140.4	137.3	137.4
5	157.8	149.6	148.9	148.7
10	182.8	174.8	172.6	169.8
15	193.5	191.6	184.9	182.4
20	202.6	201.9	196.7	194.2
25	207.7	200.0	198.2	196.1
30	205.5	201.3	200.7	195.9

2 Computational Results for Two-List Algorithms

The following tables display various computational measures (number of node scans, number of comparisons, number of updates) obtained by running the algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated directed networks. Notice that FIFO+FIFO behaves exactly like the one-list algorithm FIFO. The generated networks have source node $s = 1$ and every node is guaranteed to be accessible from s . A total of 30 replications were obtained for each network having $n = 100, 150, 200, 250, 500, 750, 1000$ nodes and density $\delta = \frac{m}{n} = 3, 4, 5, 10, 15, 20, 25, 30$. Also, we tabulate (a) the number of node scans per number of nodes, and (b) the number of updates per number of node scans.

2.1 Results for $n = 100$

Tables 43–45 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 100$ nodes and various densities δ . Also, Table 46 shows the number of node scans per number of nodes, and Table 47 shows the number of updates per number of node scans.

Table 44: Number of Comparisons ($n = 100$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	401.3	373.2	374.1	373.4
4	581.5	563.1	550.0	549.6
5	789.1	749.9	743.5	742.9
10	1826.1	1747.2	1722.4	1698.2
15	2902.6	2878.3	2771.6	2734.1
20	4048.7	4038.4	3932.4	3883.8
25	5199.6	5004.4	4959.0	4910.2
30	6158.2	6024.0	6005.7	5867.3

Table 45: Number of Updates ($n = 100$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	159.1	153.9	154.2	154.2
4	189.8	189.4	186.7	186.1
5	218.1	218.6	214.8	214.7
10	313.3	316.4	308.3	307.5
15	368.4	365.4	367.4	366.4
20	410.5	417.6	413.0	412.1
25	444.2	452.3	444.6	443.7
30	468.6	471.8	473.1	469.4

Table 46: Number of Node Scans per Node ($n = 100$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.34	1.25	1.25	1.25
4	1.45	1.40	1.37	1.37
5	1.58	1.50	1.49	1.49
10	1.83	1.75	1.73	1.70
15	1.93	1.92	1.85	1.82
20	2.03	2.02	1.97	1.94
25	2.08	2.00	1.98	1.96
30	2.06	2.01	2.01	1.96

Table 47: Number of Updates per Scan ($n = 100$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.19	1.23	1.24	1.24
4	1.30	1.35	1.36	1.35
5	1.38	1.46	1.44	1.44
10	1.71	1.81	1.79	1.81
15	1.90	1.91	1.99	2.01
20	2.03	2.07	2.10	2.12
25	2.14	2.26	2.24	2.26
30	2.28	2.34	2.36	2.40

Table 48: Number of Node Scans ($n = 150$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	212.3	197.5	196.4	196.4
4	228.2	219.3	213.5	213.5
5	235.8	227.1	221.9	222.0
10	287.4	277.4	268.3	267.2
15	302.9	307.6	294.2	289.1
20	313.3	316.0	302.2	297.4
25	313.5	315.6	301.8	298.9
30	334.0	323.8	313.9	310.4

Table 49: Number of Comparisons ($n = 150$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	636.4	591.6	589.6	589.8
4	909.7	874.4	852.3	852.8
5	1178.4	1132.9	1106.5	1107.9
10	2870.6	2773.2	2681.6	2669.4
15	4539.4	4613.8	4420.8	4340.5
20	6275.3	6314.3	6038.1	5952.9
25	7842.1	7894.2	7552.0	7482.0
30	10012.0	9718.9	9413.2	9306.9

2.2 Results for $n = 150$

Tables 48–50 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 150$ nodes and various densities δ . Also, Table 51 shows the number of node scans per number of nodes, and Table 52 shows the number of updates per number of node scans.

Table 50: Number of Updates ($n = 150$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	253.1	244.8	242.4	242.6
4	295.8	295.8	289.0	288.6
5	330.0	332.3	325.9	326.1
10	488.2	495.7	481.4	479.8
15	571.5	591.7	575.6	574.7
20	635.9	656.0	638.6	635.3
25	662.9	696.0	663.3	662.2
30	744.4	753.5	739.6	740.8

Table 51: Number of Node Scans per Node ($n = 150$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
4	1.52	1.46	1.42	1.42
5	1.57	1.51	1.48	1.48
10	1.92	1.85	1.79	1.78
15	2.02	2.05	1.96	1.93
20	2.09	2.11	2.01	1.98
25	2.09	2.10	2.01	1.99
30	2.23	2.16	2.09	2.07

Table 52: Number of Updates per Scan ($n = 150$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.19	1.24	1.23	1.24
4	1.30	1.35	1.35	1.35
5	1.40	1.46	1.47	1.47
10	1.70	1.79	1.79	1.80
15	1.89	1.92	1.96	1.99
20	2.03	2.08	2.11	2.14
25	2.11	2.21	2.20	2.22
30	2.23	2.33	2.36	2.39

Table 53: Number of Node Scans ($n = 200$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	286.7	263.7	265.1	265.5
4	305.4	290.4	286.8	286.9
5	318.2	302.1	298.3	297.7
10	385.3	378.2	366.3	362.5
15	407.9	403.0	386.4	383.9
20	437.6	438.5	421.2	411.2
25	444.6	445.4	429.0	424.4
30	445.8	438.6	427.8	423.2

Table 54: Number of Comparisons ($n = 200$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	858.9	789.8	793.4	795.4
4	1213.5	1152.4	1139.9	1140.8
5	1595.3	1508.8	1493.4	1492.1
10	3844.2	3773.1	3656.0	3620.9
15	6107.0	6045.8	5789.0	5754.8
20	8741.7	8762.9	8414.3	8215.5
25	11132.0	11155.0	10741.0	10629.0
30	13371.0	13154.0	12840.0	12705.0

2.3 Results for $n = 200$

Tables 53–55 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 200$ nodes and various densities δ . Also, Table 56 shows the number of node scans per number of nodes, and Table 57 shows the number of updates per number of node scans.

Table 55: Number of Updates ($n = 200$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	340.5	331.8	329.5	329.5
4	397.7	395.2	390.6	390.7
5	439.3	439.8	432.2	432.5
10	660.3	670.8	659.7	657.5
15	772.2	796.5	770.4	769.7
20	869.0	903.2	867.9	860.3
25	950.0	972.5	951.3	950.6
30	1001.7	1002.1	1003.0	1001.8

Table 56: Number of Node Scans per Node ($n = 200$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.43	1.32	1.33	1.33
4	1.53	1.45	1.43	1.43
5	1.59	1.51	1.49	1.49
10	1.93	1.89	1.83	1.81
15	2.04	2.02	1.93	1.92
20	2.19	2.19	2.11	2.06
25	2.22	2.23	2.14	2.12
30	2.23	2.19	2.14	2.12

Table 57: Number of Updates per Scan ($n = 200$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.19	1.26	1.24	1.24
4	1.30	1.36	1.36	1.36
5	1.38	1.46	1.45	1.45
10	1.71	1.77	1.80	1.81
15	1.89	1.98	1.99	2.01
20	1.99	2.06	2.06	2.09
25	2.14	2.18	2.22	2.24
30	2.25	2.28	2.34	2.37

Table 58: Number of Node Scans ($n = 250$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	377.6	353.3	353.3	353.2
4	382.4	366.4	359.0	359.8
5	430.7	402.1	395.2	393.4
10	495.2	483.8	469.6	465.0
15	548.5	535.8	522.9	517.3
20	566.4	564.6	549.1	539.9
25	554.4	556.1	540.2	526.1
30	563.4	571.5	549.5	540.5

2.4 Results for $n = 250$

Tables 58–60 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 250$ nodes and various densities δ . Also, Table 61 shows the number of node scans per number of nodes, and Table 62 shows the number of updates per number of node scans.

Table 59: Number of Comparisons ($n = 250$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1128.7	1057.3	1056.2	1056.6
4	1530.3	1469.6	1435.6	1440.3
5	2152.4	2009.7	1976.9	1969.6
10	4942.6	4831.0	4684.2	4638.9
15	8191.1	8009.4	7806.5	7728.7
20	11326.0	11306.0	10986.0	10792.0
25	13850.0	13907.0	13505.0	13159.0
30	16896.0	17123.0	16478.0	16210.0

Table 60: Number of Updates ($n = 250$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	450.2	437.5	437.0	437.0
4	499.3	500.5	489.0	491.2
5	595.6	584.0	577.3	575.9
10	850.0	866.4	846.3	841.7
15	1035.0	1060.8	1034.2	1029.6
20	1150.1	1170.5	1149.0	1146.8
25	1192.0	1213.8	1197.7	1191.6
30	1261.6	1310.9	1273.0	1266.2

Table 61: Number of Node Scans per Node ($n = 250$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.51	1.41	1.41	1.41
4	1.53	1.47	1.44	1.44
5	1.72	1.61	1.58	1.57
10	1.98	1.94	1.88	1.86
15	2.19	2.14	2.09	2.07
20	2.27	2.26	2.20	2.16
25	2.22	2.22	2.16	2.10
30	2.25	2.29	2.20	2.16

Table 62: Number of Updates per Scan ($n = 250$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.19	1.24	1.24	1.24
4	1.31	1.37	1.36	1.37
5	1.38	1.45	1.46	1.46
10	1.72	1.79	1.80	1.81
15	1.89	1.98	1.98	1.99
20	2.03	2.07	2.09	2.12
25	2.15	2.18	2.22	2.27
30	2.24	2.29	2.32	2.34

Table 63: Number of Node Scans ($n = 500$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	782.1	723.8	717.5	715.7
4	824.6	778.5	770.4	769.3
5	888.7	852.9	839.0	834.0
10	1092.5	1062.9	1026.9	1012.8
15	1127.0	1080.9	1065.7	1051.8
20	1207.9	1156.6	1141.4	1123.8
25	1218.8	1201.2	1170.2	1142.4
30	1267.4	1260.7	1214.3	1181.3

Table 64: Number of Comparisons ($n = 500$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	2341.2	2169.0	2150.6	2146.0
4	3301.8	3121.7	3089.7	3084.6
5	4426.3	4260.3	4185.2	4160.7
10	10921.0	10623.0	10255.0	10127.0
15	16905.0	16206.0	15984.0	15780.0
20	24166.0	23150.0	22844.0	22515.0
25	30465.0	30024.0	29264.0	28557.0
30	38008.0	37865.0	36452.0	35478.0

2.5 Results for $n = 500$

Tables 63–65 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 500$ nodes and various densities δ . Also, Table 66 shows the number of node scans per number of nodes, and Table 67 shows the number of updates per number of node scans.

Table 65: Number of Updates ($n = 500$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	931.9	899.8	893.3	891.1
4	1068.2	1058.0	1044.0	1042.6
5	1242.4	1246.4	1226.6	1223.5
10	1847.6	1866.7	1823.6	1817.4
15	2147.9	2167.2	2128.0	2120.0
20	2430.6	2476.0	2415.7	2409.0
25	2617.6	2678.6	2619.3	2604.5
30	2780.0	2883.1	2780.0	2761.3

Table 66: Number of Node Scans per Node ($n = 500$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.56	1.45	1.43	1.43
4	1.65	1.56	1.54	1.54
5	1.78	1.71	1.68	1.67
10	2.19	2.13	2.05	2.03
15	2.25	2.16	2.13	2.10
20	2.42	2.31	2.28	2.25
25	2.44	2.40	2.34	2.28
30	2.53	2.52	2.43	2.36

Table 67: Number of Updates per Scan ($n = 500$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.19	1.24	1.25	1.25
4	1.30	1.36	1.36	1.36
5	1.40	1.46	1.46	1.47
10	1.69	1.76	1.78	1.79
15	1.91	2.00	2.00	2.02
20	2.01	2.14	2.12	2.14
25	2.15	2.23	2.24	2.28
30	2.19	2.29	2.29	2.34

Table 68: Number of Node Scans ($n = 750$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1180.3	1096.7	1087.0	1084.2
4	1259.2	1194.9	1180.5	1176.3
5	1408.9	1336.5	1312.1	1305.9
10	1628.9	1608.1	1542.9	1527.4
15	1788.6	1743.4	1695.6	1670.6
20	1854.7	1806.4	1774.1	1742.5
25	1932.3	1884.3	1857.4	1827.2
30	1941.9	1917.3	1874.2	1827.4

2.6 Results for $n = 750$

Tables 68–70 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 750$ nodes and various densities δ . Also, Table 71 shows the number of node scans per number of nodes, and Table 72 shows the number of updates per number of node scans.

Table 69: Number of Comparisons ($n = 750$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	3543.6	3297.2	3264.1	3256.8
4	5030.3	4778.4	4718.7	4702.3
5	7057.7	6693.6	6576.3	6547.0
10	16273.0	16068.0	15412.0	15266.0
15	26837.0	26170.0	25456.0	25081.0
20	37080.0	36130.0	35469.0	34847.0
25	48300.0	47089.0	46424.0	45678.0
30	58283.0	57506.0	56229.0	54863.0

Table 70: Number of Updates ($n = 750$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1412.1	1363.0	1351.3	1350.8
4	1637.8	1624.5	1602.3	1598.9
5	1957.9	1962.4	1917.4	1913.8
10	2744.2	2825.5	2725.9	2720.8
15	3376.1	3441.3	3363.7	3350.1
20	3758.1	3869.8	3760.6	3744.6
25	4089.7	4230.0	4097.1	4085.2
30	4304.6	4435.6	4312.9	4287.1

Table 71: Number of Node Scans per Node ($n = 750$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.57	1.46	1.45	1.45
4	1.68	1.59	1.57	1.57
5	1.88	1.78	1.75	1.74
10	2.17	2.14	2.06	2.04
15	2.38	2.32	2.26	2.23
20	2.47	2.41	2.37	2.32
25	2.58	2.51	2.48	2.44
30	2.59	2.56	2.50	2.44

Table 72: Number of Updates per Scan ($n = 750$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.20	1.24	1.24	1.25
4	1.30	1.36	1.36	1.36
5	1.39	1.47	1.46	1.47
10	1.68	1.76	1.77	1.78
15	1.89	1.97	1.98	2.01
20	2.03	2.14	2.12	2.15
25	2.12	2.24	2.21	2.24
30	2.22	2.31	2.30	2.35

Table 73: Number of Node Scans ($n = 1000$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1625.4	1483.3	1484.0	1481.0
4	1752.1	1634.3	1618.8	1610.4
5	1842.6	1745.9	1720.9	1713.1
10	2249.7	2168.9	2125.1	2097.0
15	2435.4	2369.4	2310.0	2279.8
20	2567.5	2479.3	2458.0	2407.3
25	2622.4	2538.0	2499.6	2444.7
30	2629.8	2613.6	2547.5	2483.8

Table 74: Number of Comparisons ($n = 1000$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	4878.6	4452.5	4450.7	4445.2
4	7013.4	6533.9	6468.5	6439.6
5	9204.7	8711.7	8598.9	8563.4
10	22496.0	21680.0	21250.0	20980.0
15	36508.0	35531.0	34616.0	34178.0
20	51365.0	49630.0	49171.0	48195.0
25	65599.0	63478.0	62515.0	61176.0
30	78916.0	78414.0	76428.0	74550.0

2.7 Results for $n = 1000$

Tables 73–75 give the number of node scans, the number of comparisons, and the number of updates obtained by running algorithms FIFO+FIFO, FIFO+LIFO, FIFO+PAPE, and FIFO+PRED on randomly generated networks with $n = 1000$ nodes and various densities δ . Also, Table 76 shows the number of node scans per number of nodes, and Table 77 shows the number of updates per number of node scans.

Table 75: Number of Updates ($n = 1000$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1947.1	1852.1	1856.2	1854.3
4	2277.0	2223.7	2196.4	2190.6
5	2559.5	2550.4	2512.1	2505.1
10	3777.8	3826.0	3739.1	3722.8
15	4550.9	4604.4	4530.1	4519.7
20	5231.2	5312.2	5246.8	5212.2
25	5602.2	5748.0	5593.3	5555.2
30	5831.8	6095.2	5863.9	5821.6

Table 76: Number of Node Scans per Node ($n = 1000$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.63	1.48	1.48	1.48
4	1.75	1.63	1.62	1.61
5	1.84	1.75	1.72	1.71
10	2.25	2.17	2.13	2.10
15	2.44	2.37	2.31	2.28
20	2.57	2.48	2.46	2.41
25	2.62	2.54	2.50	2.44
30	2.63	2.61	2.55	2.48

Table 77: Number of Updates per Scan ($n = 1000$)

Density	FIFO+FIFO	FIFO+LIFO	FIFO+PAPE	FIFO+PRED
3	1.20	1.25	1.25	1.25
4	1.30	1.36	1.36	1.36
5	1.39	1.46	1.46	1.46
10	1.68	1.76	1.76	1.78
15	1.87	1.94	1.96	1.98
20	2.04	2.14	2.13	2.17
25	2.14	2.26	2.24	2.27
30	2.22	2.33	2.30	2.34