Moving to a Better Point

Consider the (feasible) basis:

$$B = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 3 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \text{and } B^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 0 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

$$c_3 - \mathbf{c}_B^T B^{-1} \mathbf{a}_3 \quad = \quad 20 - [-2, 0, 34] \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad = \quad -12$$

Since the reduced cost for x_3 is negative (equal to –12), we'd like to increase the nonbasic variable x_3 from zero.

$$B = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 3 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \text{and } B^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 0 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

Substituting in the expression for x_B gives

$$\begin{aligned} x_1 &= 1 \quad +x_3 \quad -2x_4 \quad -x_5 \geq 0 \\ x_2 &= 2 \quad -2x_3 \quad +x_4 \quad +x_5 \geq 0 \\ x_6 &= 2 \quad +x_3 \quad -2x_4 \quad -x_5 \geq 0 \end{aligned}$$

Let's increase x_3 from zero, keeping the other nonbasic variables x_4, x_5 at value 0.

As x_3 increases, the basic variables x_1, x_2, x_6 change. We require them to be nonnegative.

Solving these inequalities gives

$x_3 \geq -1$

$x_3 \leq 1$ → $0 \leq x_3 \leq 1$

$x_3 \geq -2$

$$B = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 3 & -1 \\ 1 & 1 & 0 \end{bmatrix}, \quad \text{and } B^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ -1 & 0 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

Substituting in the expression for x_B gives

$$\begin{array}{lclllll} x_1 & = & 1 & +x_3 & -2x_4 & -x_5 & = 2 \\ x_2 & = & 2 & -2x_3 & +x_4 & +x_5 & = 0 \\ x_6 & = & 2 & +x_3 & -2x_4 & -x_5 & = 3 \end{array}$$

We increase x_3 to 1, which will decrease z by 12.

The basic variables x_1, x_2, x_6 change:

- x_2 drops to zero and becomes nonbasic
- the new basis consists of $\{x_1, x_3, x_6\}$

We have completed a simplex pivot.

We have moved from $(x_1, x_2, x_6, x_3, x_4, x_5) = (1, 2, 2, 0, 0, 0)$
to the new point $(x_1, x_2, x_6, x_3, x_4, x_5) = (2, 0, 3, 1, 0, 0)$.

$$\begin{array}{ccc} x & & x + \alpha \bullet d \\ (1, 2, 2, 0, 0, 0) & \rightarrow & (1, 2, 2, 0, 0, 0) + 1\,(1, -2, 1, 1, 0\ 0) \\ & & = (2, 0, 3, 1, 0, 0). \end{array}$$

A Variation

What if the expression for x_B were

$$\begin{aligned} x_1 &= 1 + x_3 - 2x_4 - x_5 \geq 0 \\ x_2 &= 2 + 2x_3 + x_4 + x_5 \geq 0 \\ x_6 &= 2 + x_3 - 2x_4 - x_5 \geq 0 \end{aligned}$$

Now we could increase x_3 indefinitely. Since the reduced cost of x_3 is –12, we have detected an unbounded LP!