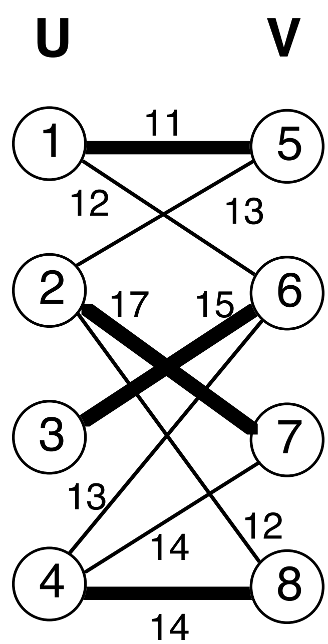
OPTIMAL ASSIGNMENT PROBLEM

An *assignment* in a network whose node set partitioned into disjoint sets U and V is a set of edges no two of which meet at a common node. Often a cost c_{uv} is associated with each possible edge (u,v), where $u \in U$ and $v \in V$.

Suppose that $|U| = |V|$. We want to determine a minimum cost assignment between the sets U and V.



Application to personnel assignment: filling jobs (V) with applicants (U), relative to training costs.

This is a minimum cost flow problem (directed arcs from U to V); each $u \in U$ has $b_u = 1$, while each $v \in V$ has $b_v = -1$.

Assignment Problem

$$\begin{bmatrix} 4 & 6 & 5 & 3 & 2 & 7 & 3 & 2 \\ 9 & 6 & 4 & 9 & 3 & 2 & 6 & 2 \\ 3 & 8 & 2 & 1 & 3 & 2 & 4 & 6 \\ 5 & 6 & 3 & 1 & 9 & 4 & 3 & 2 \\ 1 & 1 & 6 & 7 & 5 & 8 & 9 & 0 \\ 7 & 6 & 2 & 8 & 6 & 2 & 1 & 3 \\ 9 & 3 & 5 & 1 & 2 & 7 & 8 & 3 \\ 6 & 6 & 3 & 8 & 6 & 2 & 1 & 3 \end{bmatrix}$$

Find an assignment of applicants to jobs that minimizes the total cost of the assignment.

Is there an **efficient** way to solve this?

Assignment Problem

$$\begin{bmatrix} 4 & 6 & 5 & 3 & 2 & 7 & 3 & 2 \\ 9 & 6 & 4 & 9 & 3 & 2 & 6 & 2 \\ 3 & 8 & 2 & 1 & 3 & 2 & 4 & 6 \\ 5 & 6 & 3 & 1 & 9 & 4 & 3 & 2 \\ 1 & 1 & 6 & 7 & 5 & 8 & 9 & 0 \\ 7 & 6 & 2 & 8 & 6 & 2 & 1 & 3 \\ 9 & 3 & 5 & 1 & 2 & 7 & 8 & 3 \\ 6 & 6 & 3 & 8 & 6 & 2 & 1 & 3 \end{bmatrix}$$

Here is an assignment of total cost 14.

This is in fact an optimal solution to the problem.