Blending Problem

A refinery blends four petroleum components into three grades of gasoline — *regular*, *premium*, and *low lead*. The problem is to determine the optimal usage of the four components that will maximize profit.

The availabilities of components and their costs are:

component	availability barrels/day	cost $/barrel
#1	5,000	$9.00
#2	2,400	7.00
#3	4,000	12.00
#4	1,500	6.00

Blending formulas and selling price for each grade are:

grade		specification	selling price/barrel
regular (R)	(1)	not less than 40% of #1	$12.00
	(2)	not more than 20% of #2	
	(3)	not less than 30% of #3	
premium (P)	(4)	not less than 40% of #3	$18.00
low lead (L)	(5)	not more than 50% of #2	$10.00
	(6)	not less than 10% of #1	

FORMULATION:

<u>Decisions</u>: amount of each component used in each grade

<u>Variables</u>:

x_{ij} = barrels of component i used in grade j per day
(i = 1, 2, 3, 4 and j = R, P, L)

<u>Objective function</u>:

$$\max\ z = 12\,(x_{1R} + x_{2R} + x_{3R} + x_{4R}) + 18\,(x_{1P} + x_{2P} + x_{3P} + x_{4P})$$

$$+ 10\,(x_{1L} + x_{2L} + x_{3L} + x_{4L})$$

$$-\ 9\,(x_{1R} + x_{1P} + x_{1L}) - 7\,(x_{2R} + x_{2P} + x_{2L})$$

$$-\ 12\,(x_{3R} + x_{3P} + x_{3L}) - 6\,(x_{4R} + x_{4P} + x_{4L})$$

$$= 3x_{1R} + 5x_{2R} + 6x_{4R} + 9x_{1P} + 11x_{2P} + 6x_{3P}$$

$$+ 12x_{4P} + 1x_{1L} + 3x_{2L} - 2x_{3L} + 4x_{4L}$$

<u>Constraints</u>:

availability:

$$x_{1R} + x_{1P} + x_{1L} \le 5{,}000$$

$$x_{2R} + x_{2P} + x_{2L} \le 2{,}400$$

$$x_{3R} + x_{3P} + x_{3L} \le 4{,}000$$

$$x_{4R} + x_{4P} + x_{4L} \le 1{,}500$$

blending: (1) $x_{1R} / (x_{1R} + x_{2R} + x_{3R} + x_{4R}) \geq .40$

or $x_{1R} \geq .40(x_{1R} + x_{2R} + x_{3R} + x_{4R})$

or $.6x_{1R} - .4x_{2R} - .4x_{3R} - .4x_{4R} \geq 0$

(2) $x_{2R} / (x_{1R} + x_{2R} + x_{3R} + x_{4R}) \leq .20$

or $-.2x_{1R} + .8x_{2R} - .2x_{3R} - .2x_{4R} \leq 0$

(3) $x_{3R} / (x_{1R} + x_{2R} + x_{3R} + x_{4R}) \geq .30$

or $-.3x_{1R} - .3x_{2R} + .7x_{3R} - .3x_{4R} \geq 0$

(4) $x_{3P} / (x_{1P} + x_{2P} + x_{3P} + x_{4P}) \geq .40$

or $-.4x_{1P} - .4x_{2P} + .6x_{3P} - .4x_{4P} \geq 0$

(5) $x_{2L} / (x_{1L} + x_{2L} + x_{3L} + x_{4L}) \leq .50$

or $-.5x_{1L} + .5x_{2L} - .5x_{3L} - .5x_{4L} \leq 0$

(6) $x_{1L} / (x_{1L} + x_{2L} + x_{3L} + x_{4L}) \geq .10$

or $.9x_{1L} - .1x_{2L} - .1x_{3L} - .1x_{4L} \geq 0$

all $x_{ij} \geq 0$.