FINDING A FEASIBLE FLOW IN A NETWORK

Let \( G = (N, A) \) be a capacitated flow network, with \( b(i) \) the supply/demand at node \( i \); we assume \( \sum b(i) = 0 \).

**Feasible Flow Problem:**

Find flow \( \mathbf{x} : \sum_j x_{ij} - \sum_j x_{ji} = b(i) \) for each \( i \in N \)

\[ 0 \leq x_{ij} \leq u_{ij} \quad \text{for all } (i,j) \in A \]

1. Construct the *transformed network* \( G' \):

Introduce a source node \( s \), and a sink node \( t \).

   - if \( b(i) > 0 \), add the arc \((s, i)\) with capacity \( b(i) > 0 \);
   - if \( b(i) < 0 \), add the arc \((i, t)\) with capacity \(-b(i) > 0 \).

2. Then solve the maximum flow problem from \( s \) to \( t \) in the transformed network \( G' \).

**RESULT.** If the maximum flow in \( G' \) *saturates* all the source and sink arcs in \( G' \) then the original problem has a feasible solution; otherwise it is infeasible.
EXAMPLE 1

Find a maximum flow in $G'$:

The original flow problem has a feasible solution.
EXAMPLE 2

Find a maximum flow in $G'$:

The original flow problem has no feasible solution.