

MthSc 440 - HW #1

- ① Let x_1 = # acres planted with corn
 x_2 = # " " " wheat
 x_3 = # " " " soybeans
 x_4 = # " " " oats

$$\begin{aligned} \max z &= (.36)110x_1 + (.90)35x_2 + (.82)32x_3 + (.98)55x_4 \\ &= 39.6x_1 + 31.5x_2 + 26.24x_3 + 53.9x_4 \end{aligned}$$

$$\begin{aligned} \text{s.t.} \quad x_1 + x_2 + x_3 + x_4 &\leq 500 && \text{(total acres)} \\ x_3 &\leq 120 && \text{(subsidy)} \\ 110x_1 &\geq 10,000 && \text{(corn contract)} \\ x_2 - x_3 - x_4 &\geq 0 && \text{(wheat)} \\ x_1, x_2, x_3, x_4 &\geq 0 && \end{aligned}$$

- ② Let x_i = # kg of Ingredient i in each 1 kg bag ($i=1, \dots, 3$)

$$\min z = 0.55x_1 + 0.42x_2 + 0.38x_3$$

$$\begin{aligned} \text{s.t.} \quad 25x_1 + 45x_2 + 32x_3 &\geq 30 && \text{(protein)} \\ 11x_1 + 10x_2 + 7x_3 &\geq 8 && \text{(fat)} \\ 11x_1 + 10x_2 + 7x_3 &\leq 10 && \\ 235x_1 + 160x_2 + 190x_3 &\geq 200 && \text{(vitamin s)} \\ 12x_1 + 6x_2 + 10x_3 &\geq 10 && \text{(mineral t)} \\ x_1 + x_2 + x_3 &= 1 && \text{(1 kg bag)} \\ x_1, x_2, x_3 &\geq 0 && \end{aligned}$$

* Tell manufacturer to mix 0.235 kg of Ingredient 1, 0.020 kg of Ingredient 2, and 0.745 kg of Ingredient 3 in each 1 kg bag.

(Problem 2)

MIN 0.55 X1 + 0.42 X2 + 0.38 X3
SUBJECT TO
2) 25 X1 + 45 X2 + 32 X3 >= 30
3) 11 X1 + 10 X2 + 7 X3 >= 8
4) 11 X1 + 10 X2 + 7 X3 <= 10
5) 235 X1 + 160 X2 + 190 X3 >= 200
6) 12 X1 + 6 X2 + 10 X3 >= 10
7) X1 + X2 + X3 = 1
END

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) .420784300

VARIABLE	VALUE	REDUCED COST
X1	.235294	.000000
X2	.019608	.000000
X3	.745098	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.607843	.000000
3)	.000000	-.027059
4)	2.000000	.000000
5)	.000000	-.001373
6)	.392157	.000000
7)	.000000	.070196

NO. ITERATIONS = 3

③ Let x_{ij} = amt. of investment (\$) in Investment i at start of year j ($i=1, \dots, 3; j=1, \dots, 6$)

	1	2	3	4	5	6
1-yr	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
3-yr	x_{21}	x_{22}	x_{23}	x_{24}		
5-yr	x_{31}	x_{32}				

$$\max Z = 1.051x_{16} + 1.162x_{24} + 1.285x_{32}$$

$$\text{s.t.} \quad x_{11} + x_{21} + x_{31} = 10,000$$

$$x_{12} + x_{22} + x_{32} - 1.051x_{11} = 0$$

$$x_{13} + x_{23} - 1.051x_{12} = 0$$

$$x_{14} + x_{24} - 1.051x_{13} - 1.162x_{21} = 0$$

$$x_{15} - 1.051x_{14} - 1.162x_{22} = 0$$

$$x_{16} - 1.051x_{15} - 1.162x_{23} - 1.285x_{31} = 0$$

$$x_{ij} \geq 0$$

An optimal investment plan: place \$10,000 in 1-yr investment at year 1; then put proceeds \$10,510 in a 5-yr investment at year 2.

$$\text{Total gain} = 13,505.35 - 10,000.00 = 3,505.35$$

$$\text{(simple) annual yield} = \frac{(3505.35)/6}{10,000} = .0584 \Rightarrow 5.84\%$$

[584.23 per year]

(Problem 3)

MAX 1.051 X16 + 1.162 X24 + 1.285 X32
SUBJECT TO
2) X11 + X21 + X31 = 10000
3) X32 - 1.051 X11 + X12 + X22 = 0
4) - 1.051 X12 + X13 + X23 = 0
5) X24 - 1.162 X21 - 1.051 X13 + X14 = 0
6) - 1.162 X22 - 1.051 X14 + X15 = 0
7) X16 - 1.285 X31 - 1.162 X23 - 1.051 X15 = 0
END

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 13505.3500

VARIABLE	VALUE	REDUCED COST
X16	.000000	.000000
X24	.000000	.000000
X32	10510.000000	.000000
X11	10000.000000	.000000
X21	.000000	.000291
X31	.000000	.000000
X12	.000000	.000000
X22	.000000	.001454
X13	.000000	.001383
X23	.000000	.001383
X14	.000000	.001064
X15	.000000	.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.000000	1.350535
3)	.000000	1.285000
4)	.000000	1.222645
5)	.000000	1.162000
6)	.000000	1.104601
7)	.000000	1.051000

NO. ITERATIONS= 1