MthSc 440 - HW #2

① Let x_S = # Safari trucks produced per day

x_R = # Rover trucks produced per day

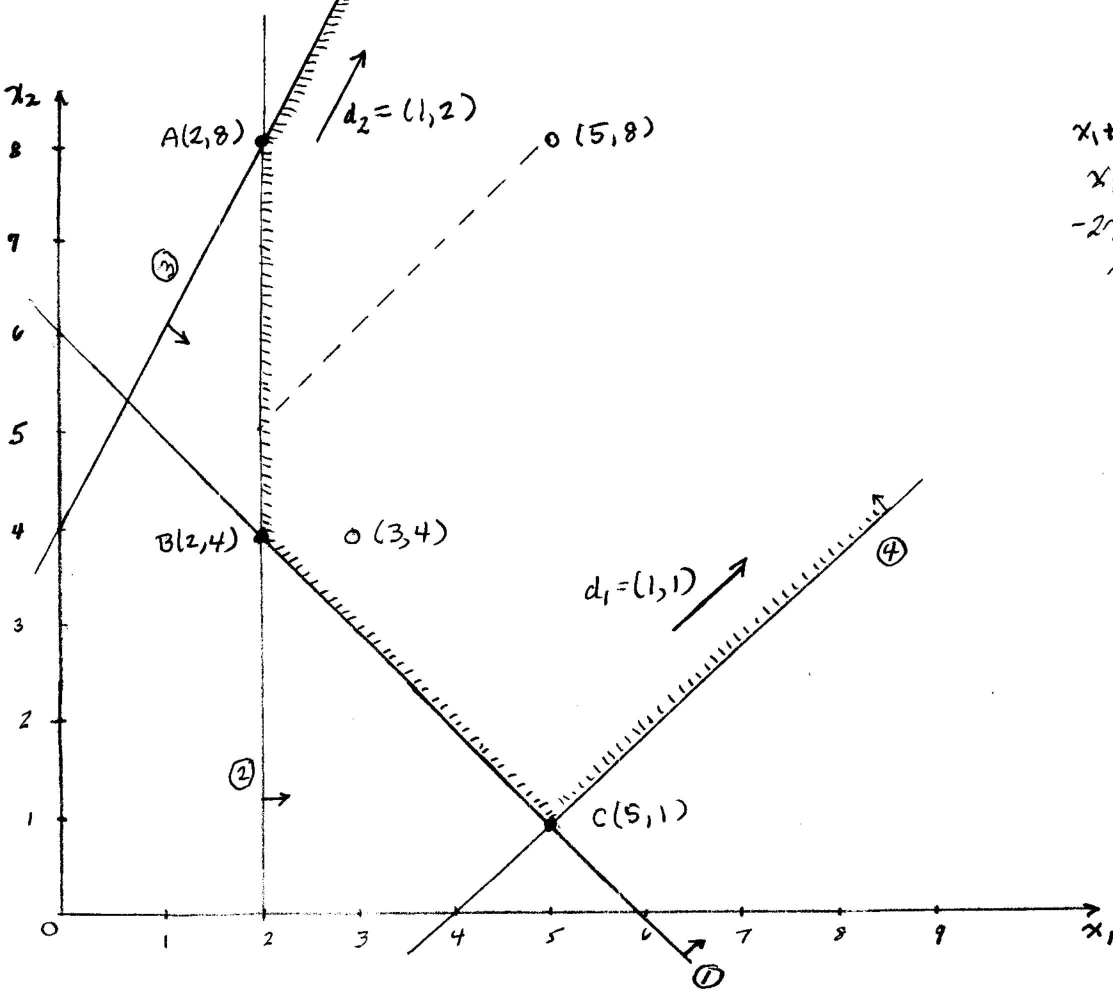
assembly: time per Safari is $\frac{1}{850}$ day, time per Rover is $\frac{1}{680}$ day.
since only have 1 day available $\Rightarrow$ $\frac{1}{850}x_S + \frac{1}{680}x_R \le 1$ or
$4x_S + 5x_R \le 3400$

painting: $\frac{1}{800}x_S + \frac{1}{700}x_R \le 1$, or $7x_S + 8x_R \le 5600$

tires: $4x_R + 5x_S \le 2900$

$$
\begin{array}{lll}
\max & z = 390x_S + 430x_R & \\
\text{s.t.} & 4x_S + 5x_R \le 3400 & \text{(assembly)} \\
 & 7x_S + 8x_R \le 5600 & \text{(painting)} \\
 & 5x_S + 4x_R \le 2900 & \text{(tires)} \\
 & x_S, x_R \ge 0 &
\end{array}
$$

(2)



$x_1 + x_2 \geq 6$ (1)
$x_1 \geq 2$ (2)
$-2x_1 + x_2 \leq 4$ (3)
$x_1 - x_2 \leq 4$ (4)
$x_1, x_2 \geq 0$

(a) extreme points: A(2,8), B(2,4), C(5,1).

extreme directions: $d_1 = (1,1)$, $d_2 = (1,2)$

(b) $(5,8) = (3,3) + (2,5) = 3 \cdot d_1 + (2,5) = \underbrace{3d_1}_{\text{nonnegative}} + \underbrace{\frac{1}{4}(2,8) + \frac{3}{4}(2,4)}_{\text{convex}}$, for example

$= 1 \cdot d_1 + 1 \cdot d_2 + \frac{1}{2}(2,8) + \frac{1}{6}(2,4) + \frac{1}{3}(5,1)$, alternatively

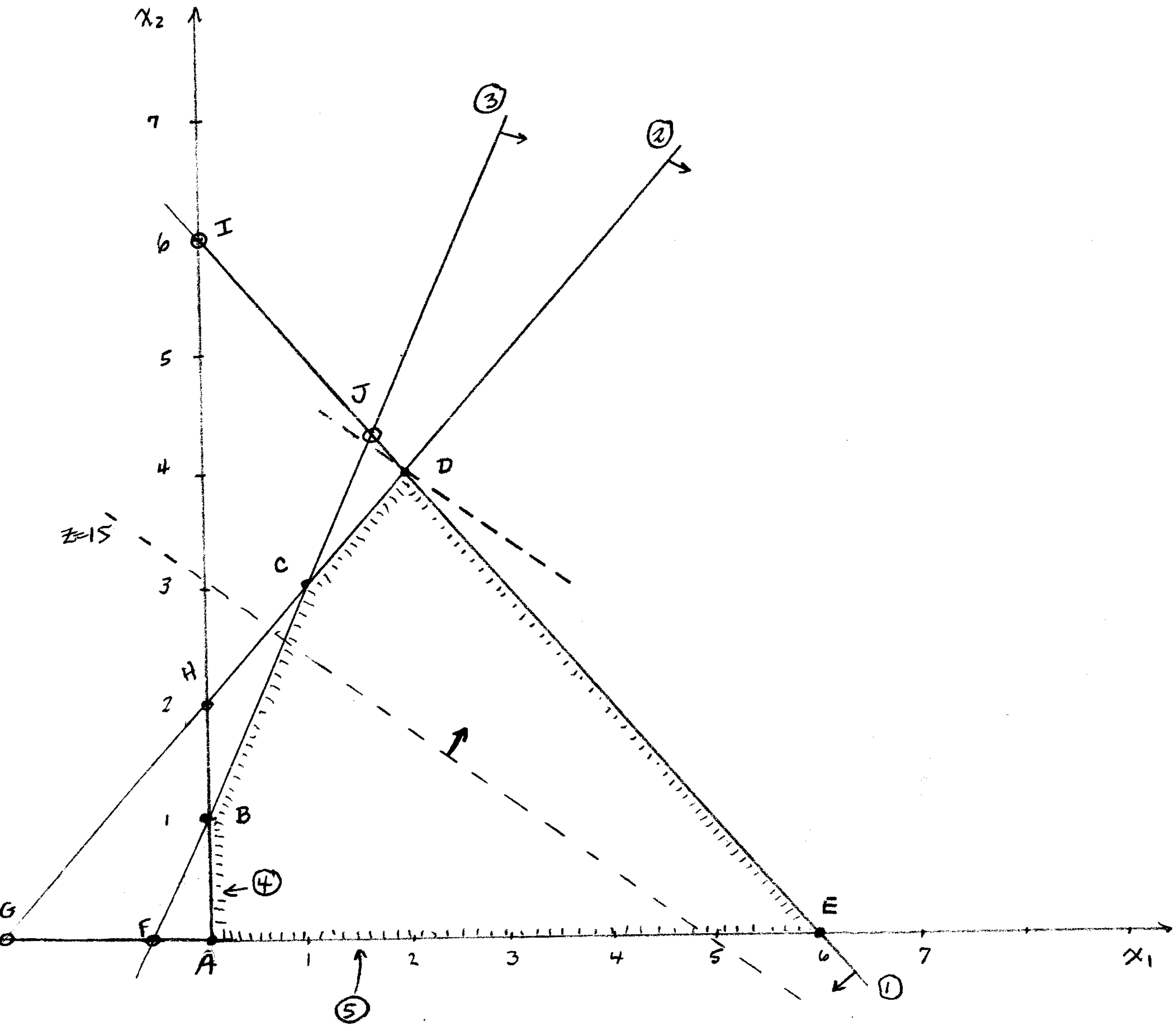
(c) $(3,4) = a(2,8) + b(2,4) + c(5,1) = (2a + 2b + 5c,\ 8a + 4b + c)$

Solve:
$$\begin{aligned} 2a + 2b + 5c &= 3 \\ 8a + 4b + c &= 4 \\ a + b + c &= 1 \end{aligned} \quad \Rightarrow \quad a = \frac{1}{4},\ b = \frac{5}{12},\ c = \frac{1}{3}$$

$$\Rightarrow \quad (3,4) = \frac{1}{4}(2,8) + \frac{5}{12}(2,4) + \frac{1}{3}(5,1)$$

3

		SLACK
$x_1 + x_2 \le 6$	1	x_3
$-x_1 + x_2 \le 2$	2	x_4
$-2x_1 + x_2 \le 1$	3	x_5
$x_1 \ge 0$	4	
$x_2 \ge 0$	5	



(a)

EP	x	basic	nonbasic	intersecting lines
A	$(0,0,6,2,1)$	x_3, x_4, x_5	x_1, x_2	4,5: $x_1=0,\ x_2=0$
B	$(0,1,5,1,0)$	x_2, x_3, x_4	x_1, x_5	3,4: $-2x_1+x_2=1,\ x_1=0$
C	$(1,3,2,0,0)$	x_1, x_2, x_3	x_4, x_5	2,3: $-x_1+x_2=2,\ -2x_1+x_2=1$
D	$(2,4,0,0,1)$	x_1, x_2, x_5	x_3, x_4	1,2: $x_1+x_2=6,\ -x_1+x_2=2$
E	$(6,0,0,8,13)$	x_1, x_4, x_5	x_2, x_3	1,5: $x_1+x_2=6,\ x_2=0$

(b) $F\left(-\frac{1}{2}, 0, \frac{13}{2}, \frac{3}{2}, 0\right)$; $G(-2,0,8,0,-3)$; $H(0,2,4,0,-1)$

$I(0,6,0,-4,-5)$; $J\left(\frac{5}{3}, \frac{13}{3}, 0, -\frac{2}{3}, 0\right)$

(c)

	A	B	C	D	E
z	0	5	18	26	18

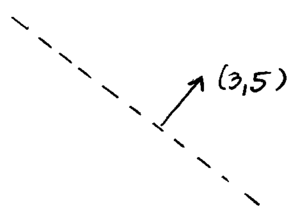
max

opt. soln is $x_1^* = 2,\ x_2^* = 4$

$z^* = 26$

(d) plot objective function line: e.g., $3x_1 + 5x_2 = 15$ (see graph)

$\nabla z = (3,5)$



move line parallel to itself, in direction of gradient, giving point D.