MthSc 440/640 Problem Set #3

Due 2/3/12

1. An automobile manufacturer produces three types of vehicles: passenger cars, two-wheel drive (2WD) trucks, and four-wheel drive (4WD) trucks. The passenger car requires 4 tires and averages 33 mpg on the highway; the 2WD truck requires 5 tires and averages 24 mpg, while the 4WD truck requires 5 tires and averages 17 mpg. Each week the manufacturer has a maximum production rate of 12,000 cars, 17,000 2WD trucks, and 14,000 4WD trucks. At most 190,000 tires can be delivered to the manufacturer each week. Net profits for each vehicle are \$310 (car), \$460 (2WD), and \$600 (4WD). Government regulations specify that the total fleet (all three types of vehicles produced) must achieve at least 23.8 mpg on average; the two types of trucks produced must together average at least 20.7 mpg. Formulate the optimal production plan as a linear programming problem in *standard inequality form*.

2. Consider the following LP problem:

$$
\begin{array}{lrl}
\min & z = & x_1 + x_2 - 4x_3 \\
\text{s.t.} & & x_1 + x_2 + 2x_3 \le 9 \\
& & x_1 + x_2 - x_3 \le 2 \\
& & -x_1 + x_2 + x_3 \le 4 \\
& & x_1, x_2, x_3 \ge 0
\end{array}
$$

(a) Convert this problem into standard equality form. Explicitly show the corresponding matrices/vectors A, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{x}$.

(b) Someone proposes $x_1 = 0, x_2 = 0, x_3 = 4$ as a solution to this problem. Verify that this is corresponds to a *basic feasible solution*, but (using our optimality criterion involving reduced costs) not an optimal solution. What is its objective function value?

(c) Continuing from the basic feasible solution in (b), use the Simplex algorithm to find an optimal solution to this problem; always choose an entering variable with the most negative reduced cost. Provide at each step the current $\mathbf{x}$-vector and objective function value z. Also indicate which variables enter and leave the basis at each step. Finally, indicate the optimal solution and objective value.

NOTE: In this problem, do not form matrix inverses; rather solve the necessary set of linear equations at each step. Show your work.