MthSc 440/640   Problem Set #4

**Due 2/8/12**

1. Consider the three vectors $v_1 = (1, -2, 3)$, $v_2 = (2, 1, 2)$, and $v_3 = (1, 2, -1)$. We want to find a *nonnegative* combination of these vectors (if one exists) that produces the vector $(1, 2, 1)$. For example using the weights 1/3, 1/6, 1/3 produces the vector $(1, 1/6, 1)$, which however is not the required vector $(1, 2, 1)$.

(a) Formulate this problem as a Phase I LP. Clearly define your variables and explicitly show the corresponding matrices/vectors $A$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{x}$.

(b) Use the Phase I method to determine a solution, if one exists, or to demonstrate that there is no solution. At each iteration show the current $\mathbf{x}$, the objective function value, as well as the entering and leaving variables. For all calculations, solve the appropriate linear systems rather than compute inverses.

2. Consider the following LP problem:

$$
\begin{array}{lrcl}
\max & z = 16x_1 + 6x_2 + 7x_3 + 4x_4 & & \\
\text{s.t.} & 4x_1 + 2x_2 + 3x_3 - x_4 & \le & 8 \\
 & x_1 + 2x_2 + 5x_3 + 5x_4 & \le & 11 \\
 & 3x_1 + x_2 + x_3 + 2x_4 & \le & 5 \\
 & x_1, x_2, x_3, x_4 & \ge & 0
\end{array}
$$

(a) Convert this problem into standard equality form.

(b) Verify that the basic variables $\{x_1, x_3, x_6\}$ define an optimal basis. Determine the exact (fractional) coordinates for the corresponding solution $\mathbf{x}$ as well as the optimal objective function value $z$.

(c) Determine the set of *all* optimal solutions $(x_1, \ldots, x_7)$ to this linear program. Express your answer in parametric form.