1. Consider the following primal LP problem (P):

\[
\begin{align*}
\text{max } z &= 3x_1 + 8x_2 + 2x_3 + 5x_4 \\
\text{s.t. } &2x_1 + x_2 + 5x_3 + 5x_4 \leq 11 \\
&x_1 + 3x_2 + 2x_3 + x_4 \leq 5 \\
&2x_1 + 4x_2 + x_3 + 3x_4 \leq 8 \\
&x_1, x_2, x_3, x_4 \geq 0
\end{align*}
\]

(a) Construct the dual problem (D).

(b) Verify that \( \mathbf{x} = (2, 1, 0, 0) \) defines a feasible solution to (P) and compute its objective function value.

(c) Compute the complementary dual vector \( \mathbf{u} = (u_1, u_2, u_3) \) and its objective function value. Is \( \mathbf{u} \) feasible? Is the \( \mathbf{x} \) in part (b) optimal? Explain.

(d) Verify that \( \mathbf{x} = (0, 7/5, 0, 4/5) \) defines a feasible solution to (P). Compute the associated complementary vector \( \mathbf{u} \) and use this to verify the optimality of \( \mathbf{x} \).

(e) Rewrite the dual (D) in standard equality form. Use this representation to help you express the objective function vector \( \mathbf{c} = (3, 8, 2, 5) \) as a nonnegative combination of the (outward pointing) normal vectors to the primal constraints that hold at \( \mathbf{x} = (0, 7/5, 0, 4/5) \). Give specific numerical values for your nonnegative weights and explicitly verify that this nonnegative combination produces \( \mathbf{c} \).