MthSc 440/640   Problem Set #5

Due 2/24/12

1. Consider the following primal LP problem (P):

$$
\begin{array}{llllll}
\max & z = 3x_1 + 8x_2 + 2x_3 + 5x_4 & & \\
\text{s.t.} & 2x_1 + x_2 + 5x_3 + 5x_4 \le 11 \\
& x_1 + 3x_2 + 2x_3 + x_4 \le 5 \\
& 2x_1 + 4x_2 + x_3 + 3x_4 \le 8 \\
& x_1, x_2, x_3, x_4 \ge 0
\end{array}
$$

(a) Construct the dual problem (D).

(b) Verify that $\mathbf{x} = (2, 1, 0, 0)$ defines a feasible solution to (P) and compute its objective function value.

(c) Compute the complementary dual vector $\mathbf{u} = (u_1, u_2, u_3)$ and its objective function value. Is $\mathbf{u}$ feasible? Is the $\mathbf{x}$ in part (b) optimal? Explain.

(d) Verify that $\mathbf{x} = (0, 7/5, 0, 4/5)$ defines a feasible solution to (P). Compute the associated complementary vector $\mathbf{u}$ and use this to verify the optimality of $\mathbf{x}$.

(e) Rewrite the dual (D) in standard equality form. Use this representation to help you express the objective function vector $\mathbf{c} = (3, 8, 2, 5)$ as a nonnegative combination of the (outward pointing) normal vectors to the primal constraints that hold at $\mathbf{x} = (0, 7/5, 0, 4/5)$. Give specific numerical values for your nonnegative weights and explicitly verify that this nonnegative combination produces $\mathbf{c}$.