Mth Sc 440 - HW#5

1 a.

PRIMAL

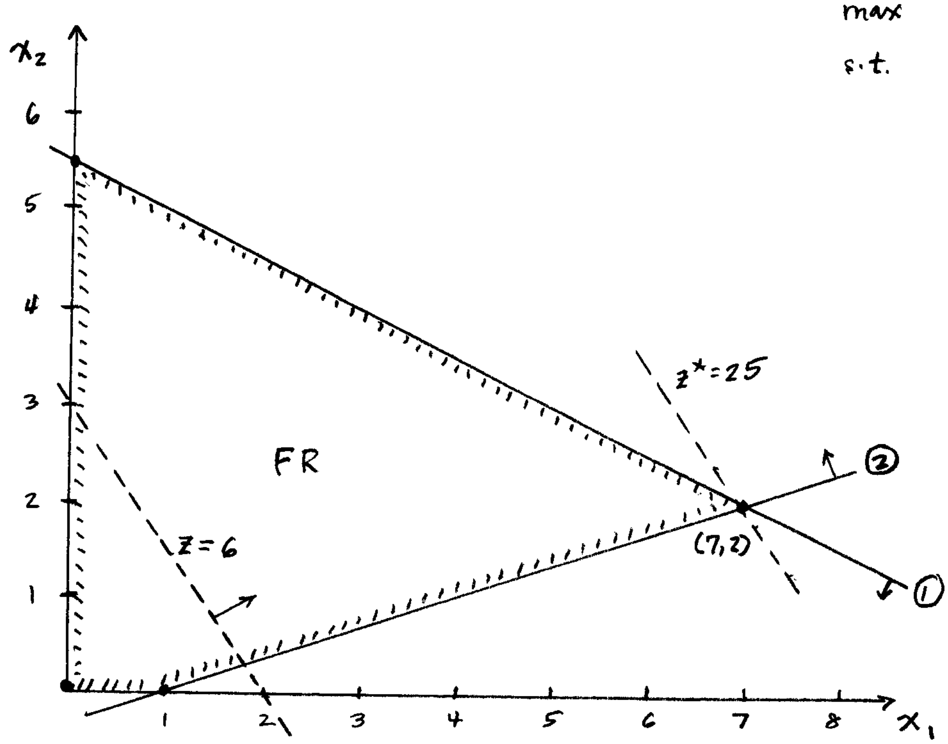
$\max \quad z = 3x_1 + 2x_2$

s.t.

$x_1 + 2x_2 \le 11$ (1)

$x_1 - 3x_2 \le 1$ (2)

$x_1, x_2 \ge 0$



optimal solution occurs at $x^* = (7,2)$ with $z^* = 25$

b.

DUAL

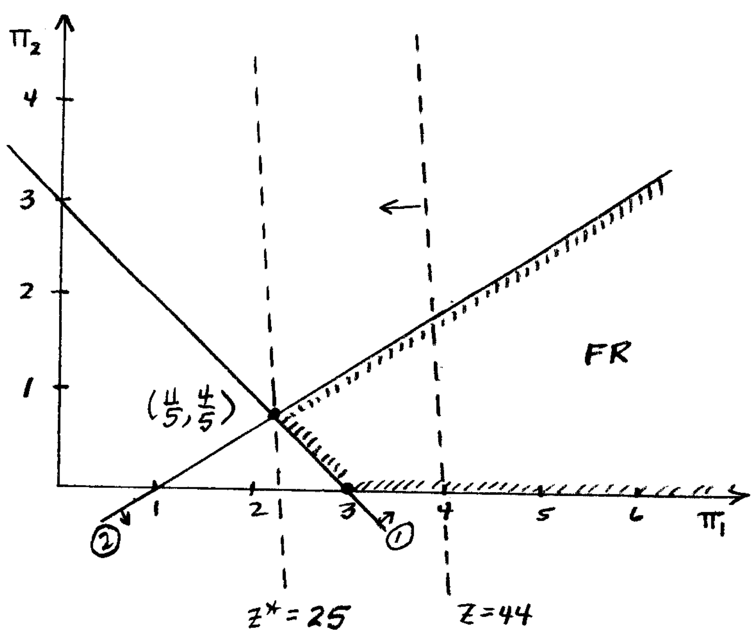
$\min \quad Z = 11\pi_1 + \pi_2$

s.t.

$\pi_1 + \pi_2 \ge 3$ (1)

$2\pi_1 - 3\pi_2 \ge 2$ (2)

$\pi_1, \pi_2 \ge 0$



optimal solution occurs at $\pi^* = (\frac{11}{5}, \frac{4}{5})$ with $Z^* = 25$

NOTE: $z^* = 25 = Z^*$
equal obj. fn. values

c. $x = (5,3)$ is feasible to primal: $5+2(3) = 11 \le 11$
$5 - 3(3) = -4 \le 1$
$x_1, x_2 \ge 0$

$z = 3(5) + 2(3) = 21$

$\pi = (3,1)$ is feasible to dual: $3+1 = 4 \ge 3$
$2(3) - 3(1) = 3 \ge 2$
$\pi_1, \pi_2 \ge 0$

$Z = 11(3) + 1 = 34$

$\boxed{z \le Z \text{ holds}}$

so WDT holds for this x, π

PRIMAL

(2) max $x_1 + 2x_2 - 3x_3$

s.t.
$$-3x_1 + x_2 + 2x_3 = 16$$
$$2x_1 + 4x_2 + 3x_3 \geq 20$$
$$x_1 \geq 0, \quad x_2 \leq 0$$
x_3 unrestricted

DUAL

min $16\pi_1 + 20\pi_2$

s.t.
$$-3\pi_1 + 2\pi_2 \geq 1$$
$$\pi_1 + 4\pi_2 \leq 2$$
$$2\pi_1 + 3\pi_2 = -3$$
π_1 unrestricted

$\pi_2 \leq 0$

PRIMAL

(3) a. max $x_1 + 2x_2 - 9x_3 + 8x_4 - 36x_5$

s.t.
$$2x_2 - x_3 + x_4 - 3x_5 \leq 40$$
$$x_1 - x_2 + 2x_4 - 2x_5 \leq 10$$
$$x_1, \ldots, x_5 \geq 0$$

DUAL

min $40\pi_1 + 10\pi_2$

$\pi_2 \geq 1$ (1)

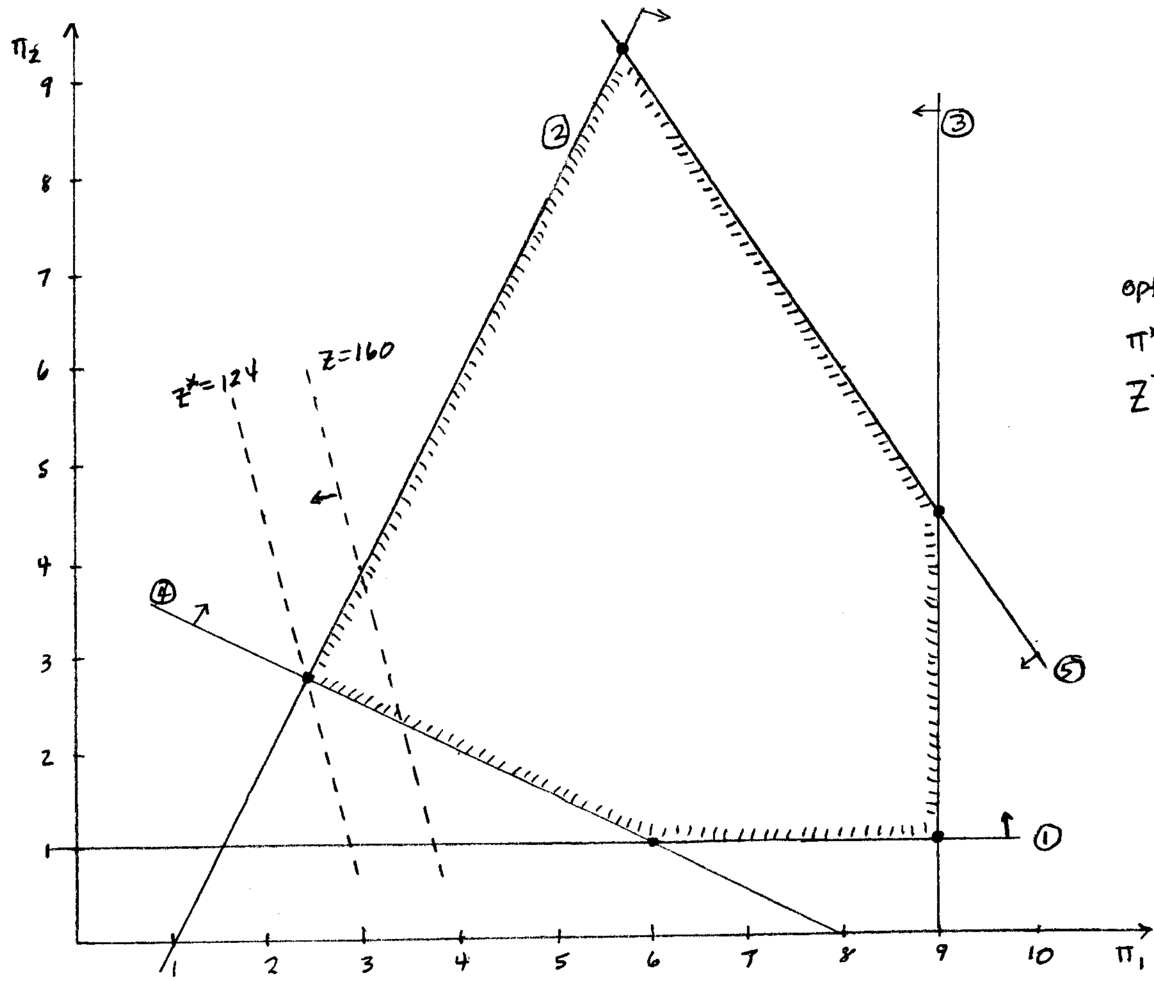
$2\pi_1 - \pi_2 \geq 2$ (2)

$-\pi_1 \geq -9$ (3)

$\pi_1 + 2\pi_2 \geq 8$ (4)

$-3\pi_1 - 2\pi_2 \geq -36$ (5)

$\pi_1, \pi_2 \geq 0$



optimal solution is $\pi^* = (\frac{12}{5}, \frac{14}{5})$ with $Z^* = 124$

③ b. For $\pi^* = (\frac{12}{5}, \frac{14}{5})$, check dual constraints:

① $\frac{14}{5} > 1 \Rightarrow x_1 = 0$

② $2(\frac{12}{5}) - \frac{14}{5} = 2$

③ $-\frac{12}{5} > -9 \Rightarrow x_3 = 0$

④ $\frac{12}{5} + 2(\frac{14}{5}) = 8$

⑤ $-3(\frac{12}{5}) - 2(\frac{14}{5}) = -\frac{64}{5} > -36 \Rightarrow x_5 = 0$

Also $\pi_1^* > 0,\ \pi_2^* > 0 \Rightarrow$

$$2x_2 - x_3 + x_4 - 3x_5 = 40$$
$$x_1 - x_2 + 2x_4 - 2x_5 = 10$$

Then
$$2x_2 + x_4 = 40$$
$$-x_2 + 2x_4 = 10$$
$$\Rightarrow x_2 = 14,\ x_4 = 12$$

$\therefore$ $x^* = (0, 14, 0, 12, 0) \geq 0$ is feasible; π^* is feasible; x^*, π^* satisfy CS $\Big\} \Rightarrow x^*$ is optimal.

[Also note: $cx^* = 2(14) + 8(12) = 124$, agreeing with $Z^* = 124$ earlier.]

PRIMAL

④ a. $\max\ z = x_1 + 2x_2 + 5x_3 + x_4$

s.t.
$$x_1 + 2x_2 + x_3 - x_4 \leq 10$$
$$-x_1 + x_2 + x_3 + x_4 \leq 12$$
$$2x_1 + x_2 + x_3 - x_4 \leq 30$$
$$x_1, x_2, x_3, x_4 \geq 0$$

DUAL

$\min\ Z = 10\pi_1 + 12\pi_2 + 30\pi_3$

s.t.
$$\pi_1 - \pi_2 + 2\pi_3 \geq 1$$
$$2\pi_1 + \pi_2 + \pi_3 \geq 2$$
$$\pi_1 + \pi_2 + \pi_3 \geq 5$$
$$-\pi_1 + \pi_2 - \pi_3 \geq 1$$
$$\pi_1, \pi_2, \pi_3 \geq 0$$

$x = (42, 0, 0, 54)$ $\quad z = 96$

$42 - 54 = -12 < 10 \Rightarrow \pi_1 = 0$

$-42 + 54 = 12$

$2(42) - 54 = 30$

$\therefore x$ is feasible

$x_1 > 0 \Rightarrow \pi_1 - \pi_2 + 2\pi_3 = 1$

$x_4 > 0 \Rightarrow -\pi_1 + \pi_2 - \pi_3 = 1$

$\Rightarrow \pi = (0, 3, 2)$

Check π for feasibility:
$0 - 3 + 4 = 1$
$0 + 3 + 2 > 2$
$0 + 3 + 2 = 5$
$-0 + 3 - 2 = 1$

$\pi = (0, 3, 2, 0, 3, 0, 0) \geq 0$
so π is feasible, and complementary to $x \Rightarrow x$ is optimal

b.

B	N	N	B	B	N	N
42	0	0	54	22	0	0
x_1	x_2	x_3	x_4	x_5	x_6	x_7
λ_1	λ_2	λ_3	λ_4	π_1	π_2	π_3
0	3	0	0	0	3	2

note: x and π are complementary

Top row of tableau is $\lambda_1 \ldots \lambda_4\ \pi_1 \ldots \pi_3$

	z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
z	1	0	3	0	0	0	3	2	96

$\therefore$ $z + 3x_2 + 3x_6 + 2x_7 = 96$; $\quad z = 96 - 3x_2 + 0x_3 - 3x_6 - 2x_7$