MthSc 440/640   Problem Set #7

Due 3/9/12

1. Consider the following LP problem (P):

$$\begin{array}{llll}
\min & z = -5x_1 + 3x_2 - 6x_3 + x_4 + 0x_5 + 0x_6 + 0x_7 \\
\text{s.t.} & 3x_1 + 2x_2 + x_3 + 2x_4 + x_5 + 0x_6 + 0x_7 = 9 \\
& x_1 + x_2 + x_3 + x_4 + 0x_5 + x_6 + 0x_7 = 4 \\
& 4x_1 + 3x_2 + 3x_3 + 4x_4 + 0x_5 + 0x_6 + x_7 = 15 \\
& x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0
\end{array}$$

(a) Let the current basis consist of $\{x_1, x_3, x_7\}$. Verify that the associated basis matrix B can be written as E_1E_2 where

$$E_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) Using the eta factorization, calculate the current basic feasible solution $\mathbf{x} = (x_1, x_2, \ldots, x_7)$.

(c) Using the eta factorization, solve for the associated dual variables u and use these to compute the reduced costs.

(d) Which variable should be chosen to enter the current basis? Determine the leaving variable by first computing d_B using the eta factorization.

(e) Determine the next eta matrix E_3 in the factorization and verify that $E_1E_2E_3$ gives the new basis matrix.

(f) Using the new eta factorization, determine whether this new basis is optimal.