MthSc 440 – Test #1

1. Let x_A = number of Desk A's manufactured per week

x_B = number of Desk B's manufactured per week

$$\max\ z = 75x_A + 52x_B - 5(8x_A + 5x_B) - 2(11x_A + 9x_B)$$
$$= 13x_A + 9x_B$$

s.t

$11x_A + 9x_B \le 980$ (lumber)

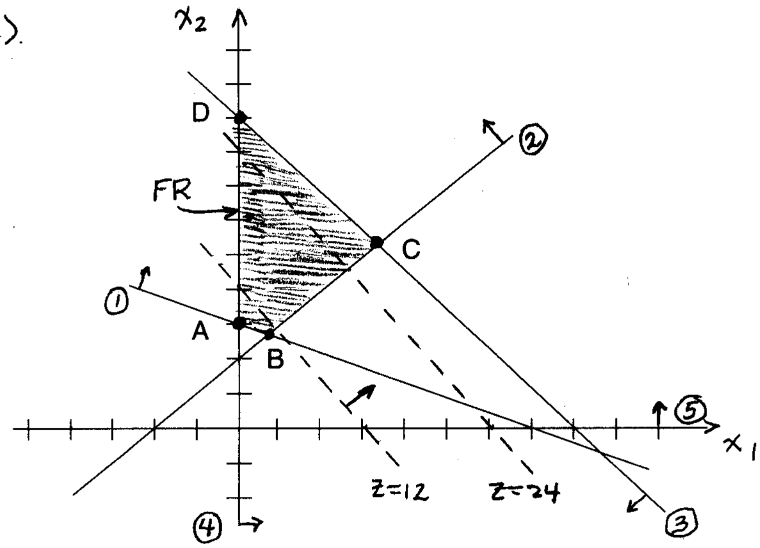
$8x_A + 5x_B \le 625$ (labor)

$0.6x_A - 0.4x_B \le 0$ (proportions)*

$x_A, x_B \geq 0$

* $\dfrac{x_A}{x_A + x_B} \le 0.4 \iff x_A \le 0.4x_A + 0.4x_B \iff 0.6x_A - 0.4x_B \le 0$

2(a).



$3x_1 + 7x_2 \geq 21$ ①
$-x_1 + x_2 \geq 2$ ②
$9x_1 + 8x_2 \leq 72$ ③
$x_1, x_2 \geq 0$ ④,⑤

(b)

Extreme point	Intersecting lines	Coordinates
A	$3x_1 + 7x_2 = 21$, $x_1 = 0$	$(0,3)$
B	$3x_1 + 7x_2 = 21$, $-x_1 + x_2 = 2$	$(\frac{7}{10}, \frac{27}{10})$
C	$-x_1 + x_2 = 2$, $9x_1 + 8x_2 = 72$	$(\frac{56}{17}, \frac{90}{17})$
D	$9x_1 + 8x_2 = 72$, $x_1 = 0$	$(0,9)$

(c) Try plotting $z = 4x_1 + 3x_2 = 12$
$z = 4x_1 + 3x_2 = 24$

Move objective function line parallel to itself in the direction of $\nabla z = (4,3)$ $\rightarrow$ reach point C. Optimal solution is $x^* = (\frac{56}{17}, \frac{90}{17})$ with $z^* = 4(\frac{56}{17}) + 3(\frac{90}{17}) = \frac{494}{17}$

(d) There are 6 basic infeasible points: intersections of lines ①,③; ①,⑤; ②,④; ②,⑤; ③,⑤; ④,⑤ 3(a) Algebraically, degeneracy means some basic variable has value 0.

Geometrically, it means that more than n hyperplanes meet at the BFS.

(b) Yes, an LP can have an optimal solution along an edge of the FR when there are alternative optimal solutions (at the extreme points at the end of the edge.

(c) Express the objective function z in terms of the current nonbasic variables $z = z_0 + a_1 x_1 + \cdots + a_n x_n$. Optimal when all $a_i \le 0$. Equivalently, when all reduced costs of nonbasic variables are ≥ 0.

(d) If all the reduced costs of the nonbasic variables are > 0, then the current (optimal) solution is unique.

4(a) $\max \quad z = 5x_1 + 6x_2 + 9x_3 + 8x_4 + 0x_5 + 0x_6$

s.t.
$$\begin{aligned} 2x_1 - x_2 + 4x_3 + 3x_4 + x_5 \quad &= 7 \\ x_1 + 2x_2 + 3x_3 - x_4 \quad + x_6 &= 5 \\ x_1, x_2, \ldots, x_6 &\geq 0 \end{aligned}$$

Need $n=4$ nonbasic variables set to zero: start at origin

$x_1 = x_2 = x_3 = x_4 = 0 \quad \Rightarrow \quad x = (0,0,0,0,7,5)$

(b)
$$\left.\begin{aligned} x_5 &= 7 - 2x_1 + x_2 - 4x_3 - 3x_4 \\ x_6 &= 5 - x_1 - 2x_2 - 3x_3 + x_4 \end{aligned}\right\} \quad \begin{aligned} x_5 &= 7 - 4x_3 \geq 0, \quad x_3 \leq 7/4 \\ x_6 &= 5 - 3x_3 \geq 0, \quad x_3 \leq 5/3 \leftarrow \text{min} \end{aligned}$$

$z = 0 + 5x_1 + 6x_2 + \underline{9}x_3 + 8x_4$ $\hookrightarrow$ so x_3 enters

so x_6 leaves

Increase x_3 to $5/3 \Rightarrow x_5 = 7 - 4(\frac{5}{3}) = \frac{1}{3}$, $x_6 = 0$, $z = 0 + 9(\frac{5}{3}) = 15$

So $x = (0, 0, \frac{5}{3}, 0, \frac{1}{3}, 0)$, $z = 15$.

(c) Select say $\{x_1, x_5\}$ to be basic: $\begin{aligned} 2x_1 + x_5 &= 7 \\ x_1 \quad &= 5 \end{aligned} \Rightarrow x_1 = 5, x_5 = -3$

$\Rightarrow x = (5,0,0,0,-3,0)$ is basic, but infeasible since $x_5 < 0$.