MthSc 441 – HW#1

(8) $P(E \cup F) = P(E) + P(F) - P(EF)$

but $P(E \cup F) \leq 1$, so $P(E) + P(F) - P(EF) \leq 1$

$\Rightarrow P(EF) \geq P(E) + P(F) - 1$

In particular, when $P(E) = 0.9$ and $P(F) = 0.8$

$$P(EF) \geq 0.9 + 0.8 - 1 = 0.7$$

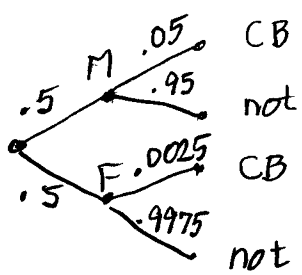
(14) Player A wins if any of following sequences of S, F (success, failure) occur: S, FFS, FFFFS, ...

Since these outcomes are mutually exclusive } say p = prob of success, $q = 1-p$ = prob of failure

$$P(A \text{ wins}) = p + q^2 p + q^4 p + \ldots$$
$$= p\left[1 + q^2 + q^4 + \ldots\right] = p\left[\frac{1}{1-q^2}\right]$$
$$= \frac{p}{(1-q)(1+q)} = \frac{1}{1+q} = \boxed{\frac{1}{2-p}}$$

$$P(B \text{ wins}) = 1 - P(A \text{ wins}) = 1 - \frac{1}{2-p} = \boxed{\frac{1-p}{2-p}}$$

(21)



$$P(M \mid CB) = \frac{P(M, CB)}{P(CB)}$$
$$= \frac{P(M) \cdot P(CB \mid M)}{P(M) \cdot P(CB \mid M) + P(F) \cdot P(CB \mid F)}$$
$$= \frac{(.5)(.05)}{(.5)(.05) + (.5)(.0025)}$$
$$\approx \boxed{0.9524}$$

(33)

	F	S	
B	4	6	10
G	6	x	$6+x$
	10	$6+x$	$16+x$

$P(B) = \frac{10}{16+x}$, $P(F) = \frac{10}{16+x}$

For independence, need

$$P(BF) = P(B)\,P(F)$$

$$\frac{4}{16+x} = \frac{10}{16+x} \cdot \frac{10}{16+x}$$

$$\Rightarrow \quad 4(16+x) = 100 \;\Rightarrow\; 16+x = 25 \;\Rightarrow\; \boxed{x = 9 \text{ sophomore girls}}$$

CHECK

	F	S	
B	4	6	10
G	6	9	15
	10	15	25

$P(BS) \stackrel{?}{=} P(B)\cdot P(S)$

$\frac{6}{25} = \frac{10}{25}\cdot\frac{15}{25} = \frac{2\cdot 3}{25}$ ✓

Similarly, $P(GF) = P(G)\cdot P(F)$

$P(GS) = P(G)\cdot P(S)$

(44)

Urn 1	Urn 2
5W	3W
7B	12B

$\xleftarrow{H}$ Urn 1 $\qquad$ Urn 2 $\xrightarrow{T}$

$$P(T|W) = \frac{P(TW)}{P(W)}$$

$$= \frac{P(T)\cdot P(W|T)}{P(T)\cdot P(W|T) + P(H)\cdot P(W|H)}$$

$$= \frac{\frac{1}{2}\cdot\frac{3}{15}}{\frac{1}{2}\cdot\frac{3}{15} + \frac{1}{2}\cdot\frac{5}{12}} = \boxed{\frac{12}{37}}$$