MthSc 441 – HW#4

④ We are interested in $P\{X_A > X_B + X_C\}$, where X_i is the service time of customer i.

a. After 10 minutes, A & B both leave and C begins service $\Rightarrow$ $P\{A \text{ is last}\} = \boxed{0}$

b. $X_A > X_B + X_C$ only when $X_A = 3$, $X_B = 1$, $X_C = 1$.

By independence, $P\{X_A=3, X_B=1, X_C=1\} =$
$P\{X_A=3\}\cdot P\{X_B=1\}\cdot P\{X_C=1\} = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \boxed{\frac{1}{27}}$

c. By lack of memory property:

$$P\{X_A > X_B + X_C\} = P\{X_A > X_B\}\cdot P\{X_A > X_C\}$$

i.e., B "beats" A and C "beats" A.

Now $P\{X_B < X_A\} = \frac{\mu_B}{\mu_A+\mu_B} = \frac{\mu}{\mu+\mu} = \frac{1}{2}$

$P\{X_C < X_A\} = \frac{\mu_C}{\mu_A+\mu_C} = \frac{\mu}{\mu+\mu} = \frac{1}{2}$

$\Rightarrow P\{A \text{ is last}\} = \frac{1}{2}\cdot\frac{1}{2} = \boxed{\frac{1}{4}}$

⑥ Say Jones currently served by Clerk 1, $\rightsquigarrow$ Expon(λ_1)
Brown currently served by Clerk 2. $\rightsquigarrow$ Expon(λ_2)

Condition on which clerk finishes first:

$P\{\text{Smith not last}\} = P\{\text{Jones completes before Brown}\}\cdot P\{\text{Smith completes before Brown} \mid J<B\} + P\{\text{Brown completes before Jones}\}\cdot P\{\text{Smith completes before Jones} \mid B<J\}$.

$$= P\{J<B\}\cdot P\{S<B\} + P\{B<J\}\cdot P\{S<J\} \leftarrow \text{by memoryless property}$$

$$= \frac{\lambda_1}{\lambda_1+\lambda_2}\cdot\frac{\lambda_1}{\lambda_1+\lambda_2} + \frac{\lambda_2}{\lambda_1+\lambda_2}\cdot\frac{\lambda_2}{\lambda_1+\lambda_2}$$

$$= \frac{\lambda_1^2+\lambda_2^2}{(\lambda_1+\lambda_2)^2}$$

(32) Time X_i spent on job i = Exponential, mean a $\rightarrow \left\{ \boxed{\begin{array}{l} E[X_i] = a \\ var[X_i] = a^2 \end{array}} \right.$

C_1 = time job 1 completed $= X_1$

$C_2 = X_1 + X_2$; $\quad C_3 = X_1 + X_2 + X_3$

$$X = C_1 + C_2 + C_3 = X_1 + (X_1 + X_2) + (X_1 + X_2 + X_3)$$
$$= 3X_1 + 2X_2 + X_3$$

$$E[X] = E[3X_1] + E[2X_2] + E[X_3] = 3a + 2a + a = \underline{6a}$$

since X_i are independent, and exponential:

$$var[X] = var[3X_1 + 2X_2 + X_3] = var[3X_1] + var[2X_2] + var[X_3]$$
$$= 9\,var[X_1] + 4\,var[X_2] + var[X_3] = 14\,var[X_1] = \underline{14a^2}$$

(57) $X(t)$ = $\underline{\text{\# of events in } (0,t)}$; $\quad P\{X(t)=i\} = e^{-\lambda t}\dfrac{(\lambda t)^i}{i!}$

a. $P\{X(1)=0\} = e^{-\lambda \cdot 1} = e^{-\lambda} = e^{-2}$

b. Want $E[T_1 + T_2 + T_3 + T_4]$ where T_i = i^{th} $\underline{\text{interarrival time}}$

$T_i \sim$ Expon(λ) = Exponential with mean $1/\lambda$

$$E[T_1 + T_2 + T_3 + T_4] = E[T_1] + E[T_2] + E[T_3] + E[T_4]$$
$$= 4\left(\frac{1}{\lambda}\right) = 2 \text{ hrs}$$

$\Rightarrow$ expect 4^{th} event to occur at 2pm.

c. $P\{X(2) \geq 2\} = 1 - P\{X(2)=0\} - P\{X(2)=1\}$

$t = 2$
$\lambda = 2$

$$= 1 - e^{-\lambda t} - (\lambda t)e^{-\lambda t}$$
$$= 1 - e^{-4} - 4e^{-4}$$
$$= 1 - 5e^{-4}$$