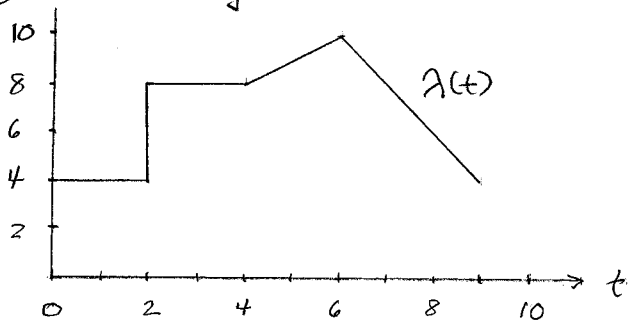


MthSc 441 - HW#5

78) Nonhomogeneous Poisson with intensity function $\lambda(t)$:



First find

$$m = \int_0^9 \lambda(t) dt$$

$$m = \int_0^2 4 dt + \int_2^4 8 dt + \int_4^6 (t+4) dt + \int_6^9 (22-2t) dt$$

$$= 8 + 16 + 18 + 21 = \boxed{63} \leftarrow \text{mean \# customers per day}$$

$$P\{i \text{ customers during day}\} = \underline{e^{-63} \frac{(63)^i}{i!}}, \text{ for } i=0,1,2,\dots$$

9) This is a pure death process, which is Poisson with parameter μ .

Can only go from state $i \rightarrow$ state j , $i \geq j$.

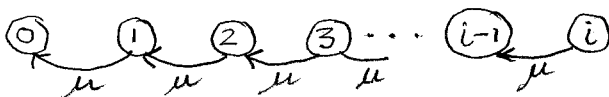
If $i \geq j > 0$: $P_{ij}(t) = P\{i-j \text{ deaths in time } t\}$

$$= \boxed{e^{-\mu t} \frac{(\mu t)^{i-j}}{(i-j)!}}$$

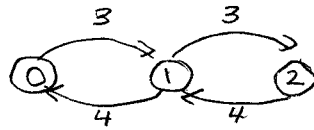
Also $P_{i0}(t) = P\{i \text{ deaths occurred by time } t\}$

$$= 1 - \sum_{j=1}^i P_{ij}(t) = 1 - \sum_{j=1}^i e^{-\mu t} \frac{(\mu t)^{i-j}}{(i-j)!}$$

$$= \boxed{1 - \sum_{k=0}^{i-1} e^{-\mu t} \frac{(\mu t)^k}{k!}}$$



(13) $X(t)$ = # of customers in shop
 $\lambda = 3/\text{hr}$, $\mu = 4/\text{hr}$



Limiting probabilities: $P_1 = \frac{3}{4} P_0$, $P_2 = \frac{9}{16} P_0$, $1 = P_0 + P_1 + P_2 \Rightarrow$
 $P_0 = \frac{16}{37}$, $P_1 = \frac{12}{37}$, $P_2 = \frac{9}{37}$

(a) avg # of customers in system = $0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2 = \frac{30}{37}$

(b) prop. of customers entering = $P(\text{not in state 2}) = 1 - P_2 = \frac{28}{37}$

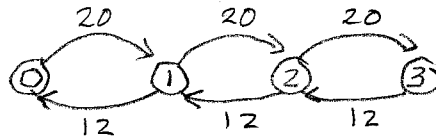
(c) $\mu = 8$: solve $P_0 = \frac{64}{97}$, $P_1 = \frac{24}{97}$, $P_2 = \frac{9}{97}$

now # customers entering per hour = $\lambda(1 - P_2) = 3 \left(\frac{88}{97}\right)$

before # customers entering per hour = $3 \left(\frac{28}{37}\right)$

\Rightarrow increase of $3 \left[\frac{88}{97} - \frac{28}{37}\right] = 0.4514$ customers/hr: 19.9% increase

(14) $X(t)$ = # cars at station
 $\lambda = 20/\text{hr}$; $\mu = 12/\text{hr}$



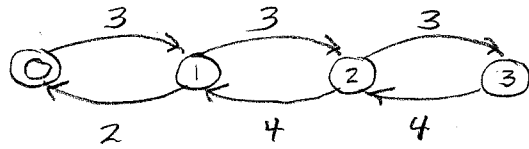
$P_1 = \frac{20}{12} P_0 = \frac{5}{3} P_0$; $P_2 = \left(\frac{5}{3}\right)^2 P_0 = \frac{25}{9} P_0$; $P_3 = \left(\frac{5}{3}\right)^3 P_0 = \frac{125}{27} P_0$

$\Rightarrow P_0 = \frac{27}{272}$, $P_1 = \frac{45}{272}$, $P_2 = \frac{75}{272}$, $P_3 = \frac{125}{272}$

(a) fraction of time busy = $1 - P_0 = \frac{245}{272}$

(b) fraction of lost customers = $P_3 = \frac{125}{272}$

⑮ $X(t)$ = number of customers in system

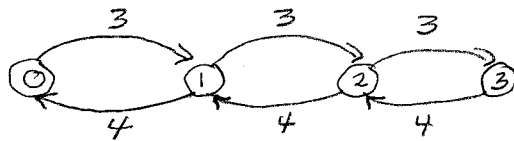


$\lambda = 3/\text{hr}$; $\mu = 2/\text{hr}$
2 servers

$$P_1 = \frac{3}{2} P_0; \quad P_2 = \frac{9}{8} P_0; \quad P_3 = \frac{27}{32} P_0 \Rightarrow P_0 = \frac{32}{143}, \dots, P_3 = \frac{27}{143}$$

(a) fraction of potential customers entering = $1 - P_3 = \frac{116}{143} \approx \underline{0.811}$

(b) Changing to a 1-server system with $\mu = 4/\text{hr}$:



$$P_1 = \frac{3}{4} P_0; \quad P_2 = \frac{9}{16} P_0; \quad P_3 = \frac{27}{64} P_0 \Rightarrow P_0 = \frac{64}{175}, \dots, P_3 = \frac{27}{175}$$

fraction of potential customers entering = $1 - P_3 = \frac{148}{175} \approx \underline{0.846}$

So this is an improvement over (a).