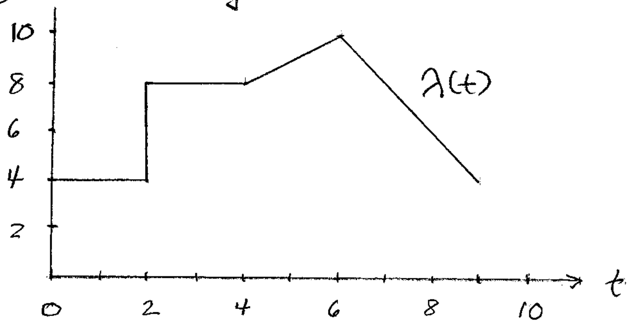
(78) Nonhomogeneous Poisson with intensity function $\lambda(t)$:



First find

$$m = \int_0^9 \lambda(t)\,dt$$

$$m = \int_0^2 4\,dt + \int_2^4 8\,dt + \int_4^6 (t+4)\,dt + \int_6^9 (22-2t)\,dt$$

$$= 8 + 16 + 18 + 21 = \boxed{63}$$ ← mean # customers per day

$P\{i \text{ customers during day}\} = \underline{e^{-63}\dfrac{(63)^i}{i!}}$, for $i = 0, 1, 2, \ldots$

(9) This is a pure death process, which is Poisson with parameter μ.

Can only go from state $i \to$ state j, $i \geq j$.

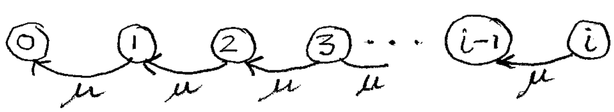
If $i \geq j > 0$: $P_{ij}(t) = P\{i-j \text{ deaths in time } t\}$

$$= \boxed{e^{-\mu t}\frac{(\mu t)^{i-j}}{(i-j)!}}$$

Also $P_{i0}(t) = P\{i \text{ deaths occurred by time } t\}$

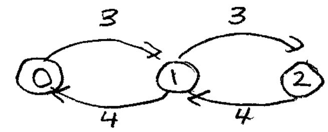
$$= 1 - \sum_{j=1}^{i} P_{ij}(t) = 1 - \sum_{j=1}^{i} e^{-\mu t}\frac{(\mu t)^{i-j}}{(i-j)!}$$

$$= \boxed{1 - \sum_{k=0}^{i-1} e^{-\mu t}\frac{(\mu t)^k}{k!}}$$



(13) $X(t) = \#$ of customers in shop

$\lambda = 3/hr, \quad \mu = 4/hr$



Limiting probabilities: $P_1 = \frac{3}{4} P_0, \quad P_2 = \frac{9}{16} P_0, \quad 1 = P_0 + P_1 + P_2 \Rightarrow$

$P_0 = \frac{16}{37}, \quad P_1 = \frac{12}{37}, \quad P_2 = \frac{9}{37}$

(a) avg # of customers in system $= 0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2 = \boxed{\frac{30}{37}}$

(b) prop. of customers entering $= P(\text{not in state } 2) = 1 - P_2 = \boxed{\frac{28}{37}}$

(c) $\mu = 8$: solve $P_0 = \frac{64}{97}, \quad P_1 = \frac{24}{97}, \quad P_2 = \frac{9}{97}$

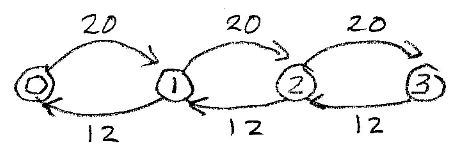
now #customers entering per hour $= \lambda(1 - P_2) = 3\left(\frac{88}{97}\right)$

before # customers entering per hour $= 3\left(\frac{28}{37}\right)$

$\Rightarrow$ increase of $3\left[\frac{88}{97} - \frac{28}{37}\right] = 0.4514$ customers/hr: 19.9% increase

(14) $X(t) = \#$ cars at station

$\lambda = 20/hr; \quad \mu = 12/hr$



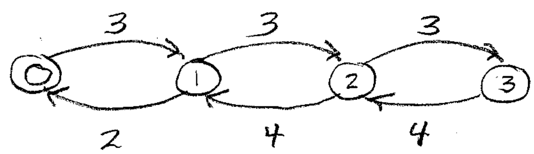
$P_1 = \frac{20}{12} P_0 = \frac{5}{3} P_0; \quad P_2 = \left(\frac{5}{3}\right)^2 P_0 = \frac{25}{9} P_0; \quad P_3 = \left(\frac{5}{3}\right)^3 P_0 = \frac{125}{27} P_0$

$\Rightarrow \quad P_0 = \frac{27}{272}, \quad P_1 = \frac{45}{272}, \quad P_2 = \frac{75}{272}, \quad P_3 = \frac{125}{272}$

(a) fraction of time busy $= 1 - P_0 = \boxed{\frac{245}{272}}$

(b) fraction of lost customers $= P_3 = \boxed{\frac{125}{272}}$

(15) $X(t)$ = number of customers in system



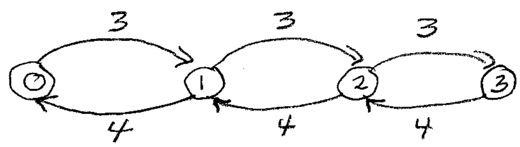
$\lambda = 3/hr$; $\mu = 2/hr$

2 servers

$$P_1 = \frac{3}{2} P_0 ; \quad P_2 = \frac{9}{8} P_0 ; \quad P_3 = \frac{27}{32} P_0 \implies P_0 = \frac{32}{143}, \ldots, P_3 = \frac{27}{143}$$

(a) fraction of potential customers entering $= 1 - P_3 = \frac{116}{143} \approx \underline{0.811}$

(b) Changing to a 1-server system with $\mu = 4/hr$:



$$P_1 = \frac{3}{4} P_0 ; \quad P_2 = \frac{9}{16} P_0 ; \quad P_3 = \frac{27}{64} P_0 \implies P_0 = \frac{64}{175}, \ldots, P_3 = \frac{27}{175}$$

fraction of potential customers entering $= 1 - P_3 = \frac{148}{175} \approx \underline{0.846}$

So this is an improvement over (a).